① Solve the differential equation of

$$\frac{dy}{dx} = (1+x^2)(1+y^2)$$

② solve $y \log y \, dx - x\,dy = 0$

③ Find general solution of differential equation $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, d$

④ Find general solution of $(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$

⑤ solve $\frac{dy}{dx} = \sqrt{4-y^2} \quad (-2<y<2)$

⑥ solve $(x+y)\frac{dy}{dx} = 1$

⑦ Find the general solution of $e^x \tan y \, dx + (1-e^x)\sec^2 y \, dy = 0$

⑧ solve $(x-y)dy - (x+y)dx = 0$

⑨ solve $\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy}$

⑩ solve $(x^2-y^2)dx + 2xy \, dy = 0$

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Solution of chapter-9 Differential Equation

① $\frac{dy}{dx} = (1+x^2)(1+y^2)$

$dy = (1+x^2)(1+y^2)dx$

$\frac{dy}{1+y^2} = (1+x^2)\,dx$

integrate $\int \frac{dy}{y^2+1^2} = \int (1+x^2)\,dx$

$\tan^{-1} y = x + \frac{x^3}{3} + C$

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② $y \log y \, dx - x\, dy = 0$

$y \log y \, dx = x\, dy$

$\frac{dx}{x} = \frac{dy}{y \log y}$

integrate

$\int \frac{1\, dx}{x} = \int \frac{\frac{1}{y}}{\log y} dy$

$\log|x| = \log(\log y) + \log C$ $\qquad \log m + \log n = \log m.n$

$\log(x) = \log C.(\log y)$

$x = C \log y$

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③ $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \tan y \, dx = -\sec^2 y \tan x \, dy$

$\frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy$

integrate $\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy$

$\log[\tan x] = -\log(\tan y) + \log C$ $\qquad \log m - \log n = \log \frac{m}{n}$

$\log(\tan x) = \log\left(\frac{C}{\tan y}\right)$

$\tan x \tan y = C$

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(4) $(e^x+e^{-x})dy-(e^x-e^{-x})dx=0$

$(e^x+e^{-x})dy=(e^x-e^{-x})dx$

$1\,dy=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}dx$

integrate $\int 1\,dy=\int\dfrac{e^x-e^{-x}}{e^x+e^{-x}}dx$

$y=\log|e^x+e^{-x}|+C$

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(5) $\dfrac{dy}{dx}=\sqrt{4-y^2}$

$\dfrac{dy}{\sqrt{2^2-y^2}}=dx$

integrate $\int\dfrac{dy}{\sqrt{2^2-y^2}}=\int 1\,dx$

$\sin^{-1}\left(\dfrac{y}{2}\right)=x+C$

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(6) $(x+y)\dfrac{dy}{dx}=1$

$\dfrac{dy}{dx}=\dfrac{1}{x+y}$

$\dfrac{dx}{dy}=x+y$

$\dfrac{dx}{dy}-x=y$

compare with $\dfrac{dy}{dx}+P(y)x=Q(y)$

$P(y)=-1 \quad Q(y)=y$

$I.F=e^{\int -1\,dy}=e^{-y}$

$x\cdot I.F=\int Q(y)\times I.F\,dy+C$

$x.e^{-y}=\int \underset{I}{y}.\underset{II}{e^{-y}}dy+C=$

$xe^{-y}=y\int e^{-y}dy-\int\left[\dfrac{d}{dy}y\int e^{-y}dy\right]dy+C$

$xe^{-y}=\dfrac{ye^{-y}}{-1}-\int 1\dfrac{e^{-y}}{-1}dy+C \Rightarrow xe^{-y}=-ye^{-y}+\int e^{-y}dy+C$

$$x e^{-y} = -y e^{-y} + \frac{e^{-y}}{-1} + c$$

$$x e^{-y} = -y e^{-y} - e^{-y} + c$$

$$x = -y - 1 + c e^{y}$$

(7) $$e^{x} \tan y \, dx + (1 - e^{x}) \sec^2 y \, dy = 0$$

$$e^{x} \tan y \, dx = -(1 - e^{x}) \sec^2 y \, dy$$

$$e^{x} \tan y \, dx = (e^{x} - 1) \sec^2 y \, dy$$

$$\frac{e^{x} \, dx}{e^{x} - 1} = \frac{\sec^2 y}{\tan y} dy$$

integrate

$$\int \frac{e^{x}}{e^{x} - 1} dx = \int \frac{\sec^2 y}{\tan y} dy$$

$$\log |e^{x} - 1| = \log |\tan y| + \log c$$

$$\log (e^{x} - 1) = \log c \cdot (\tan y)$$

$$e^{x} - 1 = c \tan y$$

(8) $$(x - y) dy - (x + y) dx = 0$$

$$(x - y) dy = (x + y) dx$$

$$\frac{dy}{dx} = \frac{x + y}{x - y} \rightarrow (1)$$

it is homogeneous equation

Put $y = vx$

Diff w.r.t x $\quad \frac{y}{x} = v$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

For eq (1)

$$v + x \frac{dv}{dx} = \frac{x + vx}{x - vx}$$

$$v + x \frac{dv}{dx} = \frac{\cancel{x}(1 + v)}{\cancel{x}(1 - v)}$$

$$x \frac{dv}{dx} = \frac{1 + v}{1 - v} - v$$

$$x\frac{dv}{dx} = \frac{1+v-v+v^2}{1-v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{1-v}$$

$$\frac{1-v}{1+v^2}\,dv = \frac{dx}{x}$$

integrate $\displaystyle\int \frac{1-v}{1+v^2}\,dv = \int \frac{dx}{x}$

$$\int \frac{1}{v^2+1^2}\,dv - \frac{1}{2}\int \frac{2v}{v^2+1}\,dv = \log|x|$$

$$\tan^{-1}v - \frac{1}{2}\log|v^2+1| = \log|x| + C$$

$$\tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left(\frac{y^2}{x^2}+1\right) = \log(x) + C$$

$$\tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left(\frac{y^2+x^2}{x^2}\right) = \log(x) + C$$

(9) $\displaystyle\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy} \text{ --- (1) } = \frac{x^2\left(1+\frac{y^2}{x^2}\right)}{x^2\left(1+\frac{xy}{x^2}\right)} = \frac{1+\left(\frac{y}{x}\right)^2}{1+\left(\frac{y}{x}\right)}$

clearly it is homogeneous equation of degree 0

Put $y = vx$ $\quad v = \frac{y}{x}$

Diff w.r.t x

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

For eq (1)

$$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{x^2+xvx}$$

$$v + x\frac{dv}{dx} = \frac{x^2(1+v^2)}{x^2(1+v)}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{1+v} - v$$

$$x\frac{dv}{dx} = \frac{1+v^2-v-v^2}{1+v}$$

$$x\,dv = \frac{1-v}{1+v}\,dx$$

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$$\int \frac{1+v}{1-v}\,dv = \int \frac{dx}{x}$$

$$-\int \frac{1+v}{v-1}\,dv = \log|x|$$

$$-\int \frac{v-1+2}{v-1}\,dv = \log|x|$$

$$-\int \left(\frac{\cancel{v-1}}{\cancel{v-1}} + \frac{2}{v-1}\right)dv = \log|x|$$

$$-\int 1\,dv - 2\int \frac{1}{v-1}\,dv = \log|x|$$

$$-v - 2\log|v-1| = \log|x| + c$$

$$-\frac{y}{x} - 2\log\left(\frac{y}{x}-1\right) = \log|x| + c$$

$$-\frac{y}{x} - 2\log\left(\frac{y-x}{x}\right) = \log|x| + c$$

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(10) $(x^2-y^2)\,dx + 2xy\,dy = 0$

$$2xy\,dy = -(x^2-y^2)\,dx$$

$$\frac{dy}{dx} = \frac{y^2-x^2}{2xy} \longrightarrow (1) \qquad \frac{\cancel{x^2}\left[\left(\frac{y}{x}\right)^2-1\right]}{\cancel{x^2}\,2\left(\frac{y}{x}\right)} = \frac{\left(\frac{y}{x}\right)^2-1}{2\left(\frac{y}{x}\right)}$$

clearly it is homogeneous equation of degree 0

Put $y = vx \qquad v = \frac{y}{x}$

Diff w.r.t x

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

For eq (1)

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2x\,vx}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{x^2}(v^2-1)}{2\cancel{x^2}v}$$

$$x\frac{dv}{dx} = \frac{v^2-1}{2v} - v$$

$$x\frac{dv}{dx} = \frac{v^2-1-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{-1-v^2}{2v}$$

$$x\,dv = \frac{-(v^2+1)}{2v}dx$$

$$\int\frac{2v}{v^2+1}dv = -\int\frac{dx}{x}$$

$$\log|v^2+1| = -\log|x| + \log c$$

$$\log\left(\frac{y^2}{x^2}+1\right) = \log\frac{c}{x}$$

$$\frac{y^2+x^2}{x^2} = \frac{c}{x} \Rightarrow \frac{y^2+x^2}{x} = c$$

$$x^2+y^2 = cx$$

which is the required differential equation