Three Dimensional Geometry

6 Marks

① Find the shortest distance between two lines

(a) $\vec{r} = \hat{i}+2\hat{j}+\hat{k}+\lambda(\hat{i}-\hat{j}+\hat{k})$

$\vec{r} = 2\hat{i}-\hat{j}-\hat{k}+\mu(2\hat{i}+\hat{j}+2\hat{k})$

(b) $\vec{r} = \hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k})$ and $\vec{r} = 2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$

(c) $\vec{r} = 6\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$ and $\vec{r} = -4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$

(d) $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

(e) $\vec{r} = \hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r} = 3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$

(f) $\vec{r} = \hat{i}+2\hat{j}+3\hat{k}+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ and $\vec{r} = 4\hat{i}+5\hat{j}+6\hat{k}+\mu(2\hat{i}+3\hat{j}+\hat{k})$

(g) $\vec{r} = \hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r} = 3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$

(h) $\vec{r} = (1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i}+(2s-1)\hat{j}-(2s-1)\hat{k}$

② Find the vector and cartesian equations of the plane that passes through the point $(1,0,-2)$ and normal to the plane is $\hat{i}+\hat{j}-\hat{k}$

③ Find the vector and cartesian equations of the plane that passes through the point $(1,4,6)$ and normal vector to the plane $\hat{i}-2\hat{j}+\hat{k}$

④ Find the vector and cartesian equation of plane which passes through point $(5,2,-4)$ and perpendicular to the line with direction ratios $(2,3,-1)$

⑤ Find the equation of plane that passes through three points $(1,1,-1)$ $(6,4,-5)$ and $(-4,-2,3)$

⑥ Find the co-ordinates of the points where the line through the points $A(3,4,1)$ and $B(5,1,6)$ crosses the (a) xy plane (b) yz plane (c) zx plane

⑦ Find the equation of the plane through the intersection of the planes $3x-y+2z-4=0$ and $x+y+z-2=0$ and the point $(2,2,1)$

⑧ Find the equation of the plane through the line of intersection of planes $x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$

⑨ Find the distance betweenofthe point $(-1,-5,-10)$ from the point of intersection of the line $\vec{r} = 2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=5$

⑩ Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x+2y-11z=3$

⑪ Find the vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot(2\hat{i}+\hat{j}+3\hat{k})=7$, $\vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k})=9$ and the point $(2,1,3)$

Solution of Three dimensional Geometry

1(a) The given lines are

$$\vec{r} = \hat{i}+2\hat{j}+\hat{k}+\lambda(\hat{i}-\hat{j}+\hat{k}) \rightarrow (1)$$
$$\vec{r} = 2\hat{i}-\hat{j}-\hat{k}+\mu(2\hat{i}+\hat{j}+2\hat{k}) \rightarrow (2)$$

compare with

$$\vec{r} = \vec{a_1}+\lambda\vec{b_1} \qquad \vec{r} = \vec{a_2}+\mu\vec{b_2}$$

$$\vec{a_1} = \hat{i}+2\hat{j}+\hat{k} \qquad \vec{b_1} = \hat{i}-\hat{j}+\hat{k}$$
$$\vec{a_2} = 2\hat{i}-\hat{j}-\hat{k} \qquad \vec{b_2} = 2\hat{i}+\hat{j}+2\hat{k}$$

$$\vec{a_2}-\vec{a_1} = 2\hat{i}-\hat{j}-\hat{k}-\hat{i}-2\hat{j}-\hat{k} = \hat{i}-3\hat{j}-2\hat{k}$$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i}(-2-1)-\hat{j}(2-2)+\hat{k}(1+2)$$
$$= -3\hat{i}+0\hat{j}+3\hat{k}$$

$$|\vec{b_1}\times\vec{b_2}| = \sqrt{(-3)^2+(0)^2+(3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$S.D = \left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right| = \left|\frac{(-3\hat{i}+0\hat{j}+3\hat{k})\cdot(\hat{i}-3\hat{j}-2\hat{k})}{3\sqrt{2}}\right|$$

$$= \left|\frac{-3+0-6}{3\sqrt{2}}\right| = \left|\frac{-\cancel{9}\,3}{\cancel{3}\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$$

1(b) The given lines are

$$\vec{r} = \hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k}) \rightarrow (1)$$
$$\vec{r} = 2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k}) \rightarrow (2)$$

Compare with

$$\vec{r} = \vec{a_1}+\lambda\vec{b_1} \qquad \vec{r} = \vec{a_2}+\mu\vec{b_2}$$

$$\vec{a_1} = \hat{i}+\hat{j} \qquad \vec{b_1} = 2\hat{i}-\hat{j}+\hat{k}$$
$$\vec{a_2} = 2\hat{i}+\hat{j}-\hat{k} \qquad \vec{b_2} = 3\hat{i}-5\hat{j}+2\hat{k}$$

$$\vec{a_2}-\vec{a_1} = 2\hat{i}+\hat{j}-\hat{k}-\hat{i}-\hat{j} = \hat{i}+0\hat{j}-\hat{k}$$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = \hat{i}(-2+5)-\hat{j}(4-3)+\hat{k}(-10+3)$$
$$= 3\hat{i}-\hat{j}-7\hat{k}$$

$$|\vec{b_1}\times\vec{b_2}| = \sqrt{(3)^2+(-1)^2+(-7)^2} = \sqrt{9+1+49} = \sqrt{59}$$

$$S.D = \left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right| = \left|\frac{(3\hat{i}-\hat{j}-7\hat{k})\cdot(\hat{i}+0\hat{j}-\hat{k})}{\sqrt{59}}\right|$$

$$SD = \left|\frac{3+0+7}{\sqrt{59}}\right| = \frac{10}{\sqrt{59}}$$

1(C) $\vec{r} = 6\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k}) \longrightarrow (1)$

$\vec{r} = -4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k}) \longrightarrow (2)$

Compare with $\vec{r} = \vec{a_1}+\lambda\vec{b_1}$ $\quad \vec{r} = \vec{a_2}+\mu\vec{b_2}$

$\vec{a_1} = 6\hat{i}+2\hat{j}+2\hat{k}$ $\quad \vec{b_1} = \hat{i}-2\hat{j}+2\hat{k}$

$\vec{a_2} = -4\hat{i}-\hat{k}$ $\quad \vec{b_2} = 3\hat{i}-2\hat{j}-2\hat{k}$

$\vec{a_2}-\vec{a_1} = -4\hat{i}-\hat{k}-6\hat{i}-2\hat{j}-2\hat{k} = -10\hat{i}-2\hat{j}-3\hat{k}$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 3 & -2 & -2\end{vmatrix} = \hat{i}(4+4)-\hat{j}(-2-6)+\hat{k}(-2+6)$$

$$= 8\hat{i}+8\hat{j}+4\hat{k}$$

$$|\vec{b_1}\times\vec{b_2}| = \sqrt{(8)^2+(8)^2+(4)^2} = \sqrt{64+64+16} = \sqrt{144} = 12$$

$$S.D = \left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right| = \left|\frac{(8\hat{i}+8\hat{j}+4\hat{k})\cdot(-10\hat{i}-2\hat{j}-3\hat{k})}{12}\right|$$

$$S.D = \left|\frac{-80-16-12}{12}\right| = \left|\frac{-\cancel{108}^{\,9}}{\cancel{12}}\right| = 9$$

1(d) The given lines are

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \Rightarrow \vec{r} = -\hat{i}-\hat{j}-\hat{k}+\lambda(7\hat{i}-6\hat{j}+\hat{k}) \rightarrow (1)$$

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \Rightarrow \vec{r} = 3\hat{i}+5\hat{j}+7\hat{k}+\mu(\hat{i}-2\hat{j}+\hat{k}) \rightarrow (2)$$

Compare with $\vec{r} = \vec{a_1}+\lambda\vec{b_1}$ $\quad \vec{r} = \vec{a_2}+\mu\vec{b_2}$

$\vec{a_1} = -\hat{i}-\hat{j}-\hat{k}$ $\quad \vec{b_1} = 7\hat{i}-6\hat{j}+\hat{k}$

$\vec{a_2} = 3\hat{i}+5\hat{j}+7\hat{k}$ $\quad \vec{b_2} = \hat{i}-2\hat{j}+\hat{k}$

$\vec{a_2}-\vec{a_1} = 3\hat{i}+5\hat{j}+7\hat{k}+\hat{i}+\hat{j}+\hat{k} = 4\hat{i}+6\hat{j}+8\hat{k}$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix} = \hat{i}(-6+2)-\hat{j}(7-1)+\hat{k}(-14+6)$$

$$= -4\hat{i}-6\hat{j}-8\hat{k}$$

$$|\vec{b_1}\times\vec{b_2}| = \sqrt{(-4)^2+(6)^2+(-8)^2} = \sqrt{16+36+64} = \sqrt{116}$$

$$S.D = \left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right| = \left|\frac{(-4\hat{i}-6\hat{j}-8\hat{k})\cdot(4\hat{i}+6\hat{j}+8\hat{k})}{\sqrt{116}}\right|$$

$$S.D = \left|\frac{-16-36-64}{\sqrt{116}}\right| = \left|\frac{-116}{\sqrt{116}}\right| = \frac{116}{\sqrt{116}} = \frac{\sqrt{116}\times\cancel{\sqrt{116}}}{\cancel{\sqrt{116}}}$$

$$S.D = \sqrt{116}$$

1(e)(i) The given lines are

$$\vec{r} = \hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k}) \rightarrow (1)$$

$$\vec{r} = 3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k}) \rightarrow (2)$$

The lines are $\parallel$ to each other because $\vec{b_1} = \vec{b_2}$

so

Compare $\vec{r} = \vec{a_1}+\lambda\vec{b}$ $\quad \vec{r} = \vec{a_2}+\mu\vec{b}$

$\vec{a_1} = (\hat{i}+2\hat{j}-4\hat{k})$ $\quad \vec{a_2} = 3\hat{i}+3\hat{j}-5\hat{k}$ $\quad \vec{b} = 2\hat{i}+3\hat{j}+6\hat{k}$

$\vec{a_2}-\vec{a_1} = 3\hat{i}+3\hat{j}-5\hat{k}-\hat{i}-2\hat{j}+4\hat{k}$

$= 2\hat{i}+\hat{j}-\hat{k}$

$$|\vec{b}| = \sqrt{(2)^2+(3)^2+(6)^2} = \sqrt{4+9+36} = \sqrt{49} = 7$$

$$S.D = \left|\frac{|\vec{b}\times(\vec{a_2}-\vec{a_1})|}{|\vec{b}|}\right| \longrightarrow (3)$$

$$\vec{b}\times(\vec{a_2}-\vec{a_1}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix} = \hat{i}(-3-6)-\hat{j}(-2-12)+\hat{k}(2-6)$$

$$\vec{b}\times(\vec{a_2}-\vec{a_1}) = -9\hat{i}+14\hat{j}-4\hat{k}$$

$$|\vec{b}\times(\vec{a_2}-\vec{a_1})| = \sqrt{(-9)^2+(14)^2+(-4)^2} = \sqrt{81+196+16} = \sqrt{293}$$

then eq (3) becomes

$$S.D = \left|\frac{\sqrt{293}}{7}\right| = \frac{\sqrt{293}}{7}$$

1(f) The given lines are

$$\vec{r} = \hat{i}+2\hat{j}+3\hat{k}+\lambda(\hat{i}-3\hat{j}+2\hat{k}) \longrightarrow (1)$$

$$\vec{r} = 4\hat{i}+5\hat{j}+6\hat{k}+\mu(2\hat{i}+3\hat{j}+\hat{k}) \longrightarrow (2)$$

compare with

$$\vec{r} = \vec{a_1}+\lambda\vec{b_1}$$

$$\vec{r} = \vec{a_2}+\mu\vec{b_2}$$

$\vec{a_1} = \hat{i}+2\hat{j}+3\hat{k}$ $\quad \vec{b_1} = \hat{i}-3\hat{j}+2\hat{k}$

$\vec{a_2} = 4\hat{i}+5\hat{j}+6\hat{k}$ $\quad \vec{b_2} = 2\hat{i}+3\hat{j}+\hat{k}$

$$\vec{a_2}-\vec{a_1} = 4\hat{i}+5\hat{j}+6\hat{k}-\hat{i}-2\hat{j}-3\hat{k}$$

$$= 3\hat{i}+3\hat{j}+3\hat{k}$$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} = \hat{i}(-3-6)-\hat{j}(1-4)+\hat{k}(3+6)$$

$$= -9\hat{i}+3\hat{j}+9\hat{k}$$

$$|\vec{b_1}\times\vec{b_2}| = \sqrt{(-9)^2+(3)^2+(9)^2} = \sqrt{81+9+81} = \sqrt{171}$$

$$S.D = \left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right| = \left|\frac{(-9\hat{i}+3\hat{j}+9\hat{k})\cdot(3\hat{i}+3\hat{j}+3\hat{k})}{\sqrt{171}}\right|$$

$$S.D = \left|\frac{\cancel{-27}+9+\cancel{27}}{\sqrt{171}}\right| = \frac{9}{\sqrt{171}}$$

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1 (h) The given lines are

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$$

$$\vec{r} = \hat{i} - t\hat{i} + t\hat{j} - 2\hat{j} + 3\hat{k} - 2t\hat{k}$$

$$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + t(-\hat{i} + \hat{j} - 2\hat{k}) \rightarrow (1)$$

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

$$\vec{r} = s\hat{i} + \hat{i} + 2s\hat{j} - \hat{j} - 2s\hat{k} - \hat{k}$$

$$\vec{r} = \hat{i} - \hat{j} - \hat{k} + s(\hat{i} + 2\hat{j} - 2\hat{k}) \rightarrow (2)$$

Compare with $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ $\quad \vec{r} = \vec{a_2} + \mu \vec{b_2}$

$\vec{a_1} = \hat{i} - 2\hat{j} + 3\hat{k}$ $\quad \vec{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$

$\vec{a_2} = \hat{i} - \hat{j} - \hat{k}$ $\quad \vec{b_2} = \hat{i} + 2\hat{j} - 2\hat{k}$

$$\vec{a_2} - \vec{a_1} = \cancel{\hat{i}} - \hat{j} - \hat{k} - \cancel{\hat{i}} + 2\hat{j} - 3\hat{k} = 0\hat{i} + \hat{j} - 4\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = \hat{i}(-2+4) - \hat{j}(2+2) + \hat{k}(-2-1)$$

$$= 2\hat{i} - 4\hat{j} - 3\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(2)^2 + (-4)^2 + (-3)^2} = \sqrt{4+16+9} = \sqrt{29}$$

$$S.D = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$= \left| \frac{(2\hat{i} - 4\hat{j} - 3\hat{k}) \cdot (0\hat{i} + \hat{j} - 4\hat{k})}{\sqrt{29}} \right|$$

$$= \left| \frac{0 - 4 + 12}{\sqrt{29}} \right| = \frac{8}{\sqrt{29}}$$

(2) Let $\vec{a}$ be the position vector of the point $(1,0,-2)$

$\vec{a} = \hat{i}+0\hat{j}-2\hat{k}$, here $\vec{n} = \hat{i}+\hat{j}-\hat{k}$

$\therefore$ Required vector equation of the plane is

Vector form $\rightarrow$

$$\vec{r}.\vec{n} = \vec{a}.\vec{n}$$

$$\vec{r}\cdot(\hat{i}+\hat{j}-\hat{k}) = (\hat{i}+0\hat{j}-2\hat{k})\cdot(\hat{i}+\hat{j}-\hat{k})$$

$$\vec{r}\cdot(\hat{i}+\hat{j}-\hat{k}) = 1+0+2$$

$$\vec{r}\cdot(\hat{i}+\hat{j}-\hat{k}) = 3 \longrightarrow (1)$$

Cartesian form $\rightarrow$ Let $\vec{r}$ be the position vector of pt $P(x,y,z)$

Put $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$ in eq (1)

$$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}+\hat{j}-\hat{k}) = 3$$

$$x+y-z = 3$$

which is required equation of plane

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(3) Let $\vec{a}$ be the position vector of the point $(1,4,6)$

$\vec{a} = \hat{i}+4\hat{j}+6\hat{k}$, here $\vec{n} = \hat{i}-2\hat{j}+\hat{k}$

$\therefore$ required vector equation of the plane is

Vector form $\rightarrow$

$$\vec{r}.\vec{n} = \vec{a}.\vec{n}$$

$$\vec{r}\cdot(\hat{i}-2\hat{j}+\hat{k}) = (\hat{i}+4\hat{j}+6\hat{k})\cdot(\hat{i}-2\hat{j}+\hat{k})$$

$$\vec{r}\cdot(\hat{i}-2\hat{j}+\hat{k}) = 1-8+6$$

$$\vec{r}\cdot(\hat{i}-2\hat{j}+\hat{k}) = -1 \longrightarrow (1)$$

Cartesian form $\rightarrow$ Let $\vec{r}$ be the position vector of pt $P(x,y,z)$

Put $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$ in eq (1)

$$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}-2\hat{j}+\hat{k}) = -1$$

$$x-2y+z = -1$$

which is required equation of plane

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(4) Let $\vec{a}$ be the position vector of the point $(5,2,-4)$

then $\vec{a} = 5\hat{i}+2\hat{j}-4\hat{k}$, here $\vec{n} = 2\hat{i}+3\hat{j}-\hat{k}$

$\therefore$ required vector equation of plane is

vector form

$$\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$$

$$\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k}) = (5\hat{i}+2\hat{j}-4\hat{k})\cdot(2\hat{i}+3\hat{j}-\hat{k})$$

$$\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k}) = 10+6+4$$

$$\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k}) = 20 \rightarrow (1)$$

Cartesian form → Let $\vec{r}$ be the position vector of the point $P(x,y,z)$

Put $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$ in equation (1)

$$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(2\hat{i}+3\hat{j}-\hat{k}) = 20$$

$$2x+3y-z = 20$$

which is required equation of plane

(5) The equation of the plane passing through the points $(1,1,-1)$, $(6,4,-5)$ and $(-4,-2,3)$ is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z+1 \\ 6-1 & 4-1 & -5+1 \\ -4-1 & -2-1 & 3+1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z+1 \\ 5 & 3 & -4 \\ -5 & -3 & 4 \end{vmatrix} = 0$$

taking -1 common from R_3

$$-\begin{vmatrix} x-1 & y-1 & z+1 \\ 5 & 3 & -4 \\ 5 & 3 & -4 \end{vmatrix} = 0$$

R_2 and R_3 are identical

$$0 = 0$$

$\therefore$ given points are collinear. So there are infinite number of planes passing through the given points

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(6) Equation of line through pts $(5,1,6)$ and $(3,4,1)$ is given by

$$\frac{x-5}{3-5} = \frac{y-1}{4-1} = \frac{z-6}{1-6}$$

$$\frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5} \longrightarrow (1)$$

(a) it crosses the xy plane it means $z=0$

$$\frac{x-5}{-2} = \frac{y-1}{3} = \frac{0-6}{-5}$$

$$\frac{x-5}{-2} = \frac{y-1}{3} = \frac{-6}{-5}$$

$$\frac{x-5}{-2} = \frac{\cancel{+}6}{\cancel{+}5} \qquad \frac{y-1}{3} = \frac{\cancel{+}6}{\cancel{+}5}$$

$$5x-25 = -12 \qquad 5y-5 = 18$$

$$5x = 13 \qquad 5y = 23$$

$$x = \frac{13}{5} \qquad y = \frac{23}{5}$$

Hence required pt $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$

(b) it crosses the yz plane it means $x=0$ in eq (1)

$$\frac{0-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5}$$

$$\frac{y-1}{3} = \frac{5}{2} \qquad \frac{z-6}{-5} = \frac{5}{2}$$

$$2y-2 = 15 \qquad 2z-12 = -25$$

$$2y = 17 \qquad 2z = -13$$

$$y = \frac{17}{2} \qquad z = \frac{-13}{2}$$

Hence required pt $\left(0, \frac{17}{2}, -13/2\right)$

(c) it crosses the zx plane it means $y=0$ in eq (1)

$$\frac{x-5}{-2} = \frac{0-1}{3} = \frac{z-6}{-5}$$

$$\frac{x-5}{-2} = \frac{-1}{3} \qquad \frac{z-6}{-5} = \frac{-1}{3}$$

$$3x-15 = 2 \qquad 3z-18 = 5$$

$$3x = 17 \qquad 3z = 23$$

$$x = \frac{17}{3} \qquad z = \frac{23}{3}$$

Hence required pt $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$

(7) Let the required equation of the plane through the intersection of planes $3x-y+2z-4=0$ and $x+y+z-2=0$ is

$$(3x-y+2z-4)+k(x+y+z-2)=0 \quad \rightarrow (1)$$

Since it passes through the point $(2,2,1)$

$$(3(2)-2+2(1)-4)+k(\cancel{2}+2+1-\cancel{2})=0$$

$$(6-\cancel{2}+\cancel{2}-4)+k(3)=0$$

$$2+3k=0$$

$$3k=-2$$

$$k=\frac{-2}{3}$$

Putting k in eq (1)

$$(3x-y+2z-4)-\frac{2}{3}(x+y+z-2)=0$$

$$\frac{9x-3y+6z-12-2x-2y-2z+4}{3}=0$$

$$7x-5y+4z-8=0$$

---

(8) Equation of plane passing through the intersection of $x+y+z=1$ and $2x+3y+4z=5$ is

$$x+y+z-1+\lambda(2x+3y+4z-5)=0 \quad \rightarrow (1)$$

$$x+y+z-1+2\lambda x+3\lambda y+4\lambda z-5\lambda=0$$

$$(1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z-1-5\lambda=0$$

But it is perpendicular to plane $x-y+z=0$

$$\therefore \quad 1(1+2\lambda)-1(1+3\lambda)+1(1+4\lambda)=0$$

$$\cancel{1}+2\lambda-\cancel{1}-3\lambda+1+4\lambda=0$$

$$6\lambda-3\lambda+1=0$$

$$3\lambda=-1$$

$$\lambda=-\frac{1}{3}$$

Putting λ in eq (1)

$$x+y+z-1-\frac{1}{3}(2x+3y+4z-5)=0$$

$$\frac{3x+\cancel{3y}+3z-3-2x-\cancel{3y}-4z+5}{3}=0$$

$$x-z+2=0$$

which is required equation of plane

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(9) The eq of line

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$$

let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$x\hat{i} + y\hat{j} + z\hat{k} - 2\hat{i} + \hat{j} - 2\hat{k} = 3\lambda\hat{i} + 4\lambda\hat{j} + 2\lambda\hat{k}$$

$$(x-2)\hat{i} + (y+1)\hat{j} + (z-2)\hat{k} = 3\lambda\hat{i} + 4\lambda\hat{j} + 2\lambda\hat{k}$$

equating co-efficient of $\hat{i}$, $\hat{j}$ and $\hat{k}$

$x-2 = 3\lambda$ | $y+1 = 4\lambda$ | $z-2 = 2\lambda$

$\frac{x-2}{3} = \lambda$ | $\frac{y+1}{4} = \lambda$ | $\frac{z-2}{2} = \lambda$

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = \lambda \longrightarrow (1)$$

and $x - y + z = 5 \longrightarrow (2)$

The coordinates of any point on line (1) are $P(2+3\lambda, -1+4\lambda, 2+2\lambda)$ if it lies on the plane (2) then

$$(2+3\lambda)(1) + (-1+4\lambda)(-1) + (2+2\lambda)(1) = 5$$

$$2 + 3\lambda + 1 - 4\lambda + 2 + 2\lambda = 5$$

$$\lambda + 5 = 5$$

$$\lambda = 0$$

$\therefore$ coordinate of point of intersection of line and plane is $P(2+0, -1+0, 2+0)$ ie $P(2,-1,2)$

Now required distance is the distance between points $(-1,-5,-10)$ and $(2,-1,2)$

$$\text{Required distance} = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$$

$$= \sqrt{(3)^2 + (4)^2 + (12)^2}$$

$$= \sqrt{9 + 16 + 144}$$

$$= \sqrt{169}$$

$$= 13$$

(10) Let θ be the angle between the line and the normal to the plane
Converting the given equation into vector form

$$\vec{r} = (-\hat{i}+3\hat{k}) + \lambda(2\hat{i}+3\hat{j}+6\hat{k})$$

and $\vec{r}\cdot(10\hat{i}+2\hat{j}-11\hat{k}) = 3$

Compare with $\vec{r} = \vec{a}+\lambda\vec{b}$ and $\vec{r}\cdot\vec{n} = d$

$\vec{b} = 2\hat{i}+3\hat{j}+6\hat{k}$ $\quad \vec{n} = 10\hat{i}+2\hat{j}-11\hat{k}$

$$\sin\theta = \left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}\right|$$

$$\sin\theta = \left|\frac{(2\hat{i}+3\hat{j}+6\hat{k})\cdot(10\hat{i}+2\hat{j}-11\hat{k})}{\sqrt{4+9+36}\,\sqrt{100+4+121}}\right|$$

$$\sin\theta = \left|\frac{20+6-66}{\sqrt{49}\,\sqrt{225}}\right|$$

$$\sin\theta = \left|\frac{-\cancel{40}\,8}{7\times\cancel{15}\,3}\right| = \left|\frac{-8}{21}\right|$$

$$\theta = \sin^{-1}\left(\frac{8}{21}\right)$$

---

(11) on comparing given planes with $\vec{r}\cdot\vec{n_1} = d_1$ and $\vec{r}\cdot\vec{n_2} = d_2$

$\vec{n_1} = 2\hat{i}+\hat{j}+3\hat{k}$ $\quad d_1 = 7$

$\vec{n_2} = 2\hat{i}+5\hat{j}+3\hat{k}$ $\quad d_2 = 9$

Required equation of plane is

$$\vec{r}\cdot(\vec{n_1}+\lambda\vec{n_2}) = d_1+\lambda d_2$$

$$\vec{r}\cdot\left(2\hat{i}+\hat{j}+3\hat{k}+\lambda(2\hat{i}+5\hat{j}+3\hat{k})\right) = 7+9\lambda \rightarrow (1)$$

$$\vec{r}\cdot\left((2+2\lambda)\hat{i}+(1+5\lambda)\hat{j}+(3+3\lambda)\hat{k}\right) = 7+9\lambda$$

Put $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$$(x\hat{i}+y\hat{j}+z\hat{k})\cdot\left((2+2\lambda)\hat{i}+(1+5\lambda)\hat{j}+(3+3\lambda)\hat{k}\right) = 7+9\lambda$$

$$(2+2\lambda)x+(1+5\lambda)y+(3+3\lambda)z = 7+9\lambda$$

the pt $(2,1,3)$ passes through given plane

$$(2+2\lambda)(2)+(1+5\lambda)(1)+(3+3\lambda)(3) = 7+9\lambda$$

$$4+4\lambda+1+5\lambda+9+\cancel{9\lambda} = 7+\cancel{9\lambda}$$

$$9\lambda+14 = 7$$

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$$9\lambda = 7-14$$

$$\lambda = -\frac{7}{9}$$

Putting λ in eq (1)

$$\vec{r}\cdot\left[2\hat{i}+\hat{j}+3\hat{k}-\frac{7}{9}(2\hat{i}+5\hat{j}+3\hat{k})\right] = 7+9\left(\frac{-7}{9}\right)$$

$$\vec{r}\cdot\left[\frac{18\hat{i}+9\hat{j}+27\hat{k}-14\hat{i}-35\hat{j}-21\hat{k}}{9}\right] = 7-7$$

$$\vec{r}\cdot\left[\frac{4\hat{i}-26\hat{j}+6\hat{k}}{9}\right] = 0$$

$$\vec{r}\cdot(4\hat{i}-26\hat{j}+6\hat{k}) = 0$$

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