(1) Find the area bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and x axis in the 1st quadrant

(2) Find the area bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(3) Find the area bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

(4) Find the area bounded by the parabola $y = x^2$ and $y = |x|$

(5) Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$

(6) Find the area of circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$

(7) Find the area bounded by curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$

(8) using Integration find the area bounded by the triangle whose vertices are $(-1,0)$ $(1,3)$ and $(3,2)$

(9) using Integration find the area of triangular region whose sides have the equations $y = 2x+1$, $y = 3x+1$ and $x = 4$

(10) Find the area enclosed between the parabola $y^2 = 4ax$ and the line $y = mx$

(11) Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$

(12) Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$

(13) Find the area bounded by the curves $\{(x,y) : y \geq x^2 \text{ and } y = |x|\}$

(14) using Integration, find the area of triangle whose vertices are $A(2,0)$ $B(4,5)$ $C(6,3)$

(15) using Integration find the area by the lines $2x+y=4$, $3x-2y=6$, $x-3y+5=0$

(16) Find the area of the region $\{(x,y) : y^2 \leq 4x,\ 4x^2+4y^2 \leq 9\}$

(17) Find the area of the region enclosed between the two curves $x^2+y^2 = 4$ and $(x-2)^2 + y^2 = 4$

(18) Find the area of the region bounded by the two parabola $y = x^2$ and $y^2 = x$

(19) Find the area of circle $x^2+y^2=16$ exterior to the parabola $y^2 = 6x$

(20) Find the area of the parabola $y^2 = 4ax$ bounded by its latus rectum

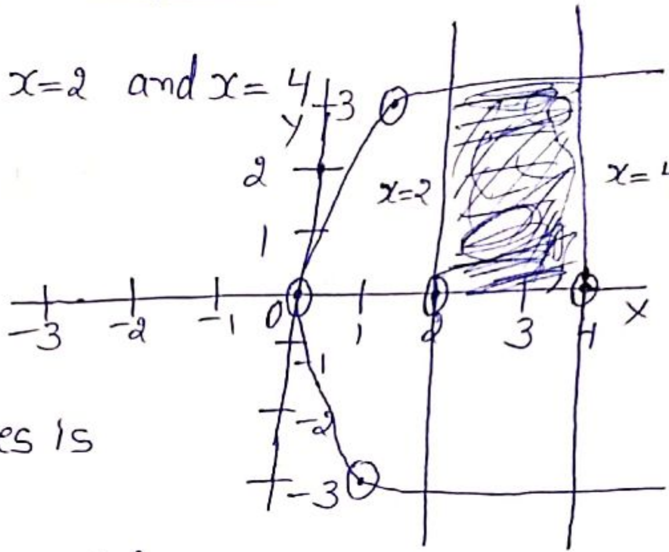
chapter – 8
Application of Integrals

(1) The eq of curves

$y^2 = 9x \Rightarrow y = 3\sqrt{x}$ $x=2$ and $x=4$

Put $x=0$
$y=0$
Put $x=1$
$y^2=9$ $y=\pm 3$

x	0	1
y	0	± 3



The area bounded by given curves is the shaded area

Required Area $= \int_2^4 y\,dx$ for parabola

$$= \int_2^4 3\sqrt{x}\,dx = 3\int_2^4 x^{\frac{1}{2}}dx$$

$$= \cancel{3}\times 2\frac{x^{3/2}}{\cancel{3}}\Big]_2^4 = 2\left[(4)^{3/2} - (2)^{3/2}\right] = 2\left[(2^2)^{3/2} - (2)^{1+\frac{1}{2}}\right]$$

$$= 2\left[8 - 2\sqrt{2}\right]$$

$$= 16 - 4\sqrt{2}$$

---

(2) The eq of ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

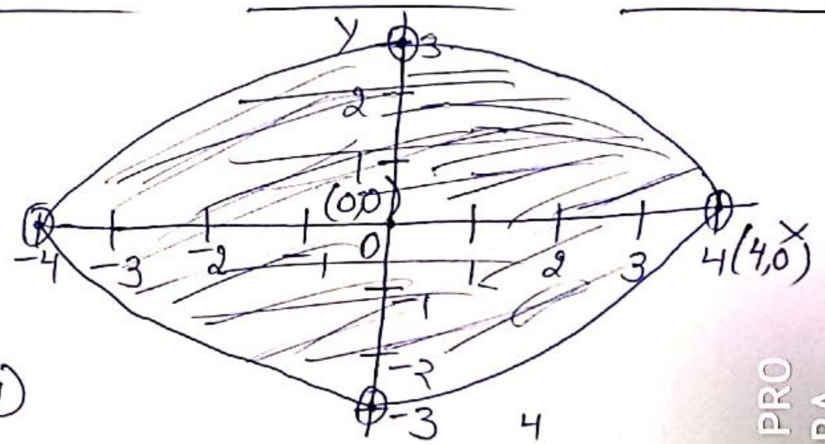
$$\frac{y^2}{9} = 1 - \frac{x^2}{16}$$

$$y^2 = \frac{9}{16}(16 - x^2)$$

$$y = \pm\frac{3}{4}\sqrt{4^2 - x^2} \rightarrow (1)$$

Put $x = 0$
$y = \frac{3}{\cancel{4}}\times\cancel{4} = \pm 3$
Put $x = \pm 4$
$y = 0$

x	0	± 4
y	± 3	0



Area of shaded region $= 4\int_0^4 y\,dx$

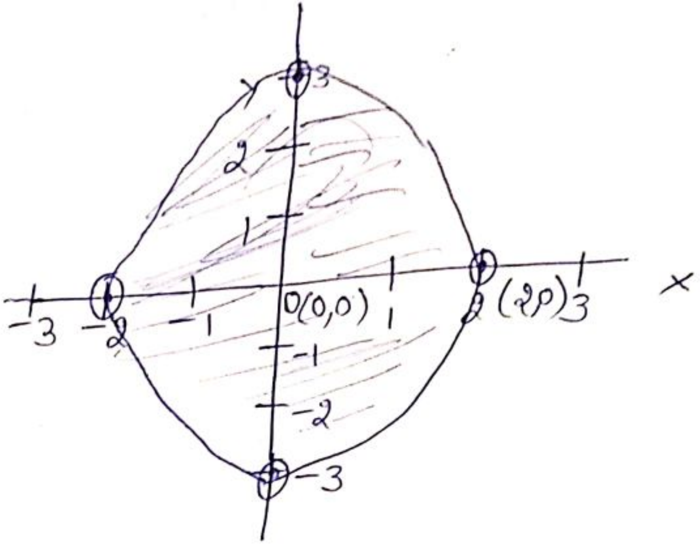
$$= \cancel{4}\int_0^4 \frac{3}{\cancel{4}}\sqrt{4^2 - x^2}\,dx$$

$$= 3\left[\frac{x}{2}\sqrt{4^2 - x^2} + \frac{(4)^2}{2}\sin^{-1}\frac{x}{4}\right]_0^4$$

$$= 3\left[\left(\frac{4}{2}\times 0 + \frac{16}{2}\sin^{-1}1\right) - \left(0 + \frac{16}{2}\sin^{-1}0\right)\right] = 3\left[0 + 8\times\frac{\pi}{2}\right]$$

$$= 12\pi$$

Q→3 The eq of ellipse



$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

$$\frac{y^2}{9}=1-\frac{x^2}{4}$$

$$y^2=\frac{9}{4}(4-x^2)$$

$$y=\pm\frac{3}{2}\sqrt{2^2-x^2}$$

Put $x=0$

$$y=\pm\frac{3}{2}\times 2=\pm 3$$

Put $x=2$

$y=0$

x	0	± 2
y	± 3	0

Area of shaded region $= 4\int_0^2 \frac{3}{2}\sqrt{2^2-x^2}\,dx$

$$=6\left[\frac{x}{2}\sqrt{2^2-x^2}+\frac{2^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$=6\left[\left(0+2\sin^{-1}\left(\frac{2}{2}\right)\right)-\left(0+2\sin^{-1}0\right)\right]$$

$$=6\left[2\times\sin^{-1}1\right]$$

$$=\cancel{12}^{6}\times\frac{\pi}{\cancel{2}}=6\pi$$

---

(4) The given curves are

$y=x^2 \rightarrow (1)$

when $x=0$

$y=0.$

when $y=1$

$x^2=1$

$x=\pm 1$

x	0	± 1
y	0	1

$y=|x| \rightarrow (2)$

when $x=-2$

$y=2$

when $x=-1$

$y=1$

when $x=0$

$y=0$

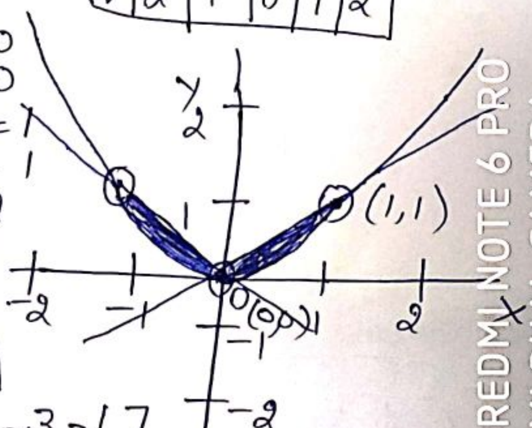
when $x=1$

$y=1$

when $x=2$

$y=2$

x	-2	-1	0	1	2
y	2	1	0	1	2



Area of shaded region $=2\left[\int_0^1 y\,dx \text{ for line} - \int_0^1 y\,dx \text{ for parabola}\right]$

$$=2\left[\int_0^1 x\,dx-\int_0^1 x^2\,dx\right]\Rightarrow 2\left[\frac{x^2}{2}\right]_0^1-\left[\frac{x^3}{3}\right]_0^1$$

$$=2\left[\left(\frac{1}{2}-0\right)-\left(\frac{1}{3}-0\right)\right]=2\left[\frac{1}{2}-\frac{1}{3}\right]=2\left[\frac{3-2}{6}\right]=\cancel{2}\left[\frac{1}{\cancel{6}3}\right]=\frac{1}{3}$$

(5) The given curves are

$x^2 = 4y \rightarrow (1)$

Put $y=0$ $\quad y = \frac{x^2}{4}$

$x = 0$

Put $y = 1$

$x^2 = 4$

$x = \pm 2$

x	0	± 2
y	0	1

$x = 4y - 2 \rightarrow (2)$

Put $y=0$ $\quad x+2 = 4y$

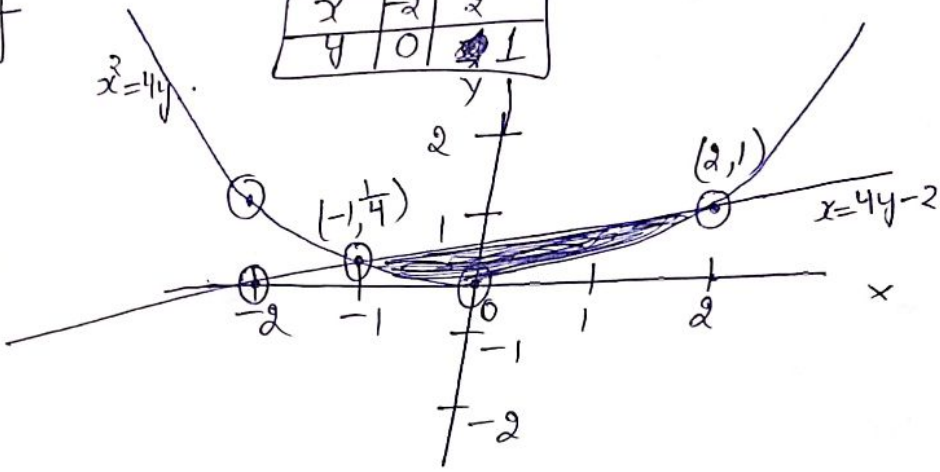
$x = -2$ $\quad y = \frac{x+2}{4}$

Put $y = 1$

$x = 4-2$

$x = 2$

x	-2	2
y	0	1



solve eq (1) and (2)

$(4y-2)^2 = 4y$

$16y^2 + 4 - 16y - 4y = 0$

$16y^2 - 20y + 4 = 0$

$4y^2 - 5y + 1 = 0$

$4y^2 - 4y - y + 1 = 0$

$4y(y-1) - 1(y-1) = 0$

$(y-1)(4y-1) = 0$

$y = 1 \qquad y = \frac{1}{4}$

when $y=1$ then eq (2)

$x = 4-2$

$x = 2$

when $y = \frac{1}{4}$ then eq (2)

$x = 4\left(\frac{1}{4}\right) - 2$

$x = -1$

Hence the pts are $(2,1)$ and $\left(-1, \frac{1}{4}\right)$

Area of shaded region $= \int_{-1}^{2} y\,dx$ for line $- \int_{-1}^{2} y\,dx$ for parabola

$= \int_{-1}^{2} \frac{x+2}{4}\,dx - \int_{-1}^{2} \frac{x^2}{4}\,dx$

$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2}$

$= \frac{1}{4}\left[\left(\frac{4}{2}+4\right) - \left(\frac{1}{2} - 2\right)\right] - \frac{1}{12}\left[(2)^3 - (-1)^3\right]$

$= \frac{1}{4}\left[(2+4) - \left(\frac{1-4}{2}\right)\right] - \frac{1}{12}[8+1]$

$= \frac{1}{4}\left[6 + \frac{3}{2}\right] - \frac{1}{\cancel{12}_{4}}(\cancel{9})^{3} \Rightarrow \frac{1}{4}\left[\frac{15}{2}\right] - \frac{3}{4}$

$= \frac{1}{4}\left[\frac{15}{2} - 3\right] = \frac{1}{4}\left[\frac{15-6}{2}\right] = \frac{9}{8}$

(6) The eq of curves are

$4x^2+4y^2=9 \rightarrow (1)$

$x^2+y^2=\frac{9}{4}$

$x^2=\frac{9}{4}-y^2$

$x = \sqrt{\left(\frac{3}{2}\right)^2-y^2}$

Put $x=0$

$y=3/2=1.5$

Put $x=3/2$

$y=0$

x	0	3/2
y	3/2	0

$x^2=4y \rightarrow (2)$

$x=2\sqrt{y}$ $\quad y=\frac{x^2}{4}$

Put $y=0$

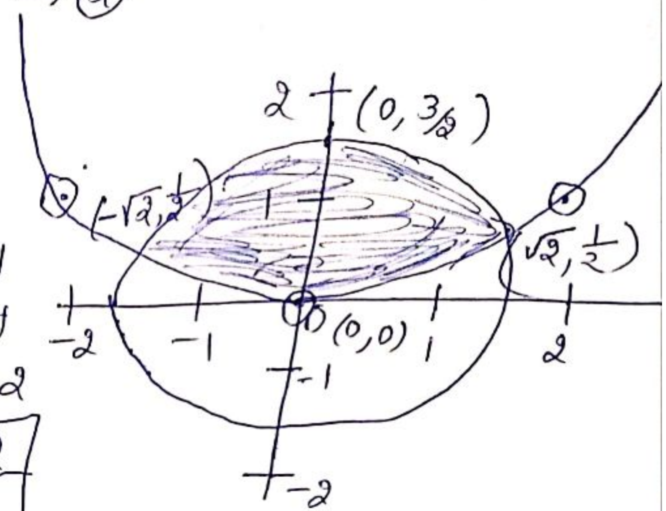
$x=0$

Put $y=1$

$x^2=4$

$x=\pm 2$

x	0	± 2
y	0	1



solve eq (1) and (2)

$4(4y)+4y^2=9$

$4y^2+16y-9=0$

$4y^2+18y-2y-9=0$

$2y(2y+9)-1(2y+9)=0$

$(2y+9)(2y-1)=0$

$y=1/2 \qquad y=-9/2$ is rejected

when $y=1/2$

then eq (2)

$x^2=4\left(\frac{1}{2}\right)$

$x=\pm\sqrt{2}$

Hence the pts of intersection are $(\sqrt{2},\frac{1}{2})$ and $(-\sqrt{2},\frac{1}{2})$

Area of shaded region $= 2\left[\underset{\text{for parabola}}{\int_0^{1/2} x\,dy} + \underset{\text{for circle}}{\int_{1/2}^{3/2} x\,dy}\right]$

$$= 2\left[\int_0^{1/2} 2\sqrt{y}\,dy + \int_{1/2}^{3/2}\sqrt{\left(\frac{3}{2}\right)^2-y^2}\,dy\right]$$

$$= 2\left[\left[\frac{2\times 2y^{3/2}}{3}\right]_0^{1/2} + \left\{\frac{y}{2}\sqrt{\left(\frac{3}{2}\right)^2-y^2}+\frac{9/4}{2}\sin^{-1}\frac{2y}{3}\right\}_{1/2}^{3/2}\right]$$

$$= 2\left[\frac{4}{3}\left(\left(\frac{1}{2}\right)^{3/2}-0^{3/2}\right) + \left\{\left(0+\frac{9}{8}\sin^{-1}1\right)-\left(\frac{1}{4}\sqrt{\frac{9}{4}-\frac{1}{4}}+\frac{9}{8}\sin^{-1}\frac{1}{3}\right)\right]$$

$$2\left[\frac{\cancel{4}^{2}}{3}\times\frac{1}{\cancel{2}\sqrt{2}}+\frac{9}{8}\times\frac{\pi}{2}-\frac{1}{4}\sqrt{\frac{8^2}{4}}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$2\left[\frac{2}{3\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}+\frac{9\pi}{16}-\frac{\sqrt{2}}{4}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$2\left[\frac{\cancel{2}\sqrt{2}}{\cancel{6}\,3}+\frac{9\pi}{16}-\frac{\sqrt{2}}{4}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$2\left[\sqrt{2}\left(\frac{1}{3}-\frac{1}{4}\right)+\frac{9}{8}\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right)\right]$$

$$2\left[\sqrt{2}\left(\frac{4-3}{12}\right)+\frac{9}{8}\cos^{-1}\frac{1}{3}\right] \qquad \boxed{\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}}$$

$$2\left[\sqrt{2}\times\frac{1}{12}+\frac{9}{8}\sin^{-1}\sqrt{1-\left(\frac{1}{3}\right)^2}\right] \qquad \boxed{\cos^{-1}x=\sin^{-1}\sqrt{1-x^2}}$$

$$2\left[\frac{\sqrt{2}}{12}+\frac{9}{8}\sin^{-1}\sqrt{1-\frac{1}{9}}\right]$$

$$2\left[\frac{\sqrt{2}}{12}+\frac{9}{8}\sin^{-1}\sqrt{\frac{8}{9}}\right]$$

$$\frac{\cancel{2}\sqrt{2}}{\cancel{12}\,6}+\frac{2\times 9}{\cancel{8}\,4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

$$\frac{\sqrt{2}}{6}+\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

---

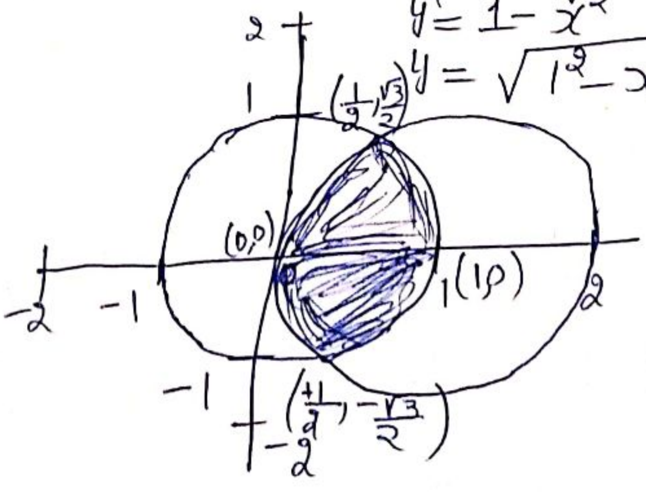
7) The given curves are

$(x-1)^2+y^2=1 \longrightarrow (1)$ $\qquad$ $x^2+y^2=1 \longrightarrow (2)$

having centre $(1,0)$ and radius 1 $\qquad$ having centre $(0,0)$ and radius

$y^2=1-(x-1)^2$ $\qquad$ $y^2=1-x^2$

$y=\sqrt{1^2-(x-1)^2}$ $\qquad$ $y=\sqrt{1^2-x^2}$



Solve eq ① and ②

$$(x-1)^2+y^2 = x^2+y^2$$
$$x^2+1-2x = x^2$$
$$-2x = -1$$
$$x = 1/2$$

when $x = 1/2$

then eq ②

$$\left(\frac{1}{2}\right)^2 + y^2 = 1 \Rightarrow y^2 = 1-\frac{1}{4} \Rightarrow y^2 = \frac{3}{4} \Rightarrow y = \pm\frac{\sqrt{3}}{2}$$

Hence pt' of intersections are $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$$\text{Required Area} = 2\left[\underset{\text{for first circle}}{\int_0^{1/2} y\,dx} + \underset{\text{for 2nd circle}}{\int_{1/2}^{1} y\,dx}\right]$$

$$= 2\left[\int_0^{1/2}\sqrt{1^2-(x-1)^2}\,dx + \int_{1/2}^{1}\sqrt{1^2-x^2}\,dx\right]$$

$$= 2\left[\left[\frac{x-1}{2}\sqrt{1^2-(x-1)^2} + \frac{1}{2}\sin^{-1}\left(\frac{x-1}{1}\right)\right]_0^{1/2} + \left[\frac{x}{2}\sqrt{1^2-x^2} + \frac{1}{2}\sin^{-1}\frac{x}{1}\right]_{1/2}^{1}\right]$$

$$= 2\left[\left\{\frac{\frac{1}{2}-1}{2}\sqrt{1^2-\left(\frac{1}{2}-1\right)^2} + \frac{1}{2}\sin^{-1}\left(\frac{1}{2}-1\right)\right\} - \left\{\frac{0-1}{2}\sqrt{1^2-(0-1)^2} + \frac{1}{2}\sin^{-1}\left(\frac{0-1}{1}\right)\right\} + \left\{\frac{1}{2}\sqrt{1-1} + \frac{1}{2}\sin^{-1}1 - \left[\frac{1}{4}\sqrt{1-\frac{1}{4}} + \frac{1}{2}\sin^{-1}\frac{1}{2}\right]\right\}\right]$$

$$= 2\left[\left\{-\frac{1}{4}\sqrt{1-\frac{1}{4}} + \frac{1}{2}\sin^{-1}\left(\frac{-1}{2}\right)\right\} - \left\{0 + \frac{1}{2}\sin^{-1}(-1)\right\} + \left\{0 + \frac{1}{2}\sin^{-1}(1) - \frac{1}{4}\sqrt{\frac{3}{4}} - \frac{1}{2}\sin^{-1}\frac{1}{2}\right\}\right]$$

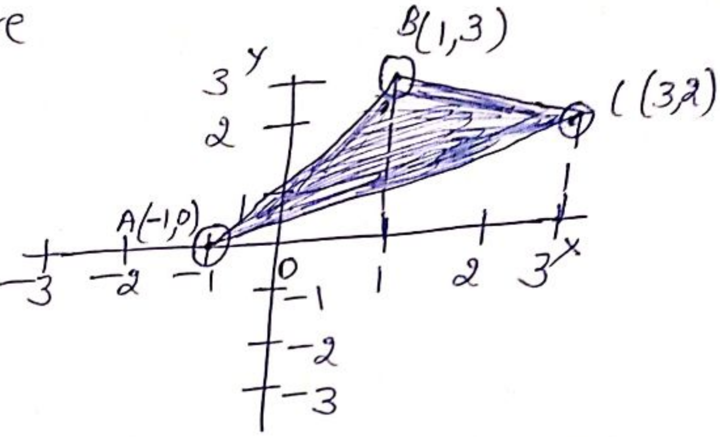
$$= 2\left[\frac{-1}{4}\frac{\sqrt{3}}{2} + \frac{1}{2}\left(\frac{-\pi}{6}\right) - \frac{1}{2}\left(\frac{-\pi}{2}\right) + \frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{1}{4}\frac{\sqrt{3}}{2} - \frac{1}{2}\left(\frac{\pi}{6}\right)\right]$$

$$= 2\left[\frac{-\sqrt{3}}{8} - \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi}{4} - \frac{\sqrt{3}}{8} - \frac{\pi}{12}\right]$$

$$= 2\left[\frac{-2\sqrt{3}}{8} - \frac{2\pi}{12} + \frac{2\pi}{4}\right] = 2\left[\frac{-\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\pi}{2}\right] = 2\left[\frac{-\sqrt{3}}{4} + \left(\frac{-\pi+3\pi}{6}\right)\right]$$

$$= 2\left[\frac{-\sqrt{3}}{4} + \frac{\cancel{2}\pi}{\cancel{6}\,3}\right] = \frac{2\pi}{3} - \frac{\cancel{2}\sqrt{3}}{\cancel{4}\,2} = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) \text{Ans}$$

(8) Let the vertices of triangle are
$A(-1,0)$ $B(1,3)$ $C(3,2)$



eq of line AB

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-0 = \frac{3-0}{1+1}(x+1)$$

$$y = \frac{3}{2}(x+1) \rightarrow (1)$$

eq of line BC

$$y-3 = \frac{2-3}{3-1}(x-1)$$

$$y-3 = \frac{-1}{2}(x-1)$$

$$2y-6 = -x+1 \Rightarrow 2y = -x+1+6 \Rightarrow y = \frac{7-x}{2}$$

eq of line AC

$$y-0 = \frac{2-0}{3+1}(x+1) \Rightarrow y = \frac{\cancel{2}^{1}}{\cancel{4}_{2}}(x+1) \Rightarrow y = \frac{x}{2}+\frac{1}{2}$$

Area of shaded region $= \int_{-1}^{1} y\,dx_{\text{ line }AB} + \int_{1}^{3} y\,dx_{\text{ line }BC} - \int_{-1}^{3} y\,dx_{\text{ line }AC}$

$$= \int_{-1}^{1} \frac{3}{2}(x+1)\,dx + \int_{1}^{3} \frac{7-x}{2}\,dx - \int_{-1}^{3}\left(\frac{x}{2}+\frac{1}{2}\right)dx$$

$$= \frac{3}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{1} + \frac{1}{2}\left[7x-\frac{x^2}{2}\right]_{1}^{3} - \frac{1}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{3}$$

$$= \left[\frac{3}{2}\left[\left(\frac{1}{2}+1\right)-\left(\frac{1}{2}-1\right)\right] + \frac{1}{2}\left[\left(21-\frac{9}{2}\right)-\left(7-\frac{1}{2}\right)\right] - \frac{1}{2}\left[\left(\frac{9}{2}+3\right)-\left(\frac{1}{2}-1\right)\right]\right]$$

$$= \frac{3}{2}\left[\frac{3}{2}+\frac{1}{2}\right] + \frac{1}{2}\left[\frac{33}{2}-\frac{13}{2}\right] - \frac{1}{2}\left[\frac{15}{2}+\frac{1}{2}\right]$$

$$= \frac{3}{\cancel{2}}\left[\frac{\cancel{4}}{\cancel{2}}\right] + \frac{1}{\cancel{2}}\left[\frac{\cancel{20}^{5}}{\cancel{2}}\right] - \frac{1}{\cancel{2}}\left[\frac{\cancel{16}^{4}}{\cancel{2}}\right]$$

$3+5-4$

$8-4$

4

⑨ The equations of sides of the triangle are

$y = 2x+1 \rightarrow (1)$ $y = 3x+1 \rightarrow (2)$ $x = 4 \rightarrow (3)$

solve eq (1) and (2)

$2x+\cancel{1} = 3x+\cancel{1}$

$x = 0$

For eq (1)

$y = 0+1 = 1$

Hence pt $A(0,1)$

solve eq (2) and (3)

$x = 4$

$y = 3(4)+1$

$y = 12+1$

$y = 13$

Hence pt $B(4,13)$

solve eq (3) and (1)

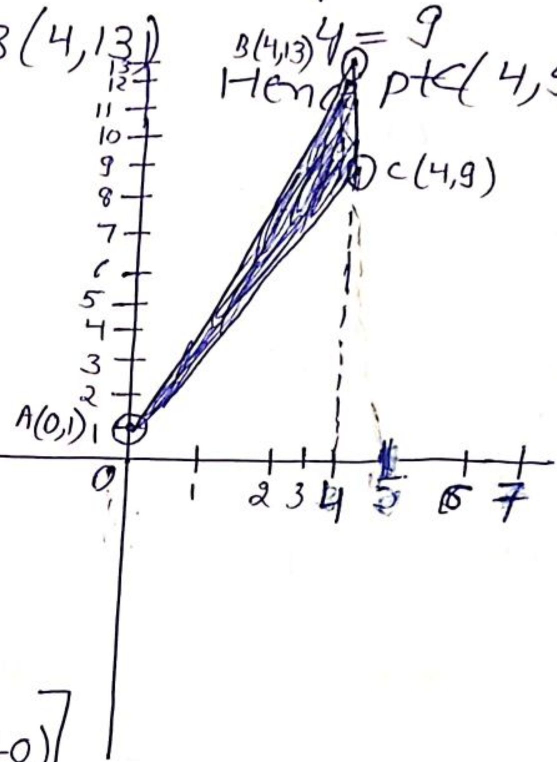
$x = 4$

$y = 2(4)+1$

$y = 8+1$

$y = 9$

Hence pt $C(4,9)$



Area of ΔABC

$$= \int_0^4 y\,dx_{\text{for line AB}} - \int_0^4 y\,dx_{\text{for line AC}}$$

$$= \int_0^4 (3x+1)\,dx - \int_0^4 (2x+1)\,dx$$

$$= \left[\frac{3x^2}{2}+x\right]_0^4 - \left[\frac{\cancel{2}x^2}{\cancel{2}}+x\right]_0^4$$

$$= \left[\left(\frac{3}{\cancel{2}}(\cancel{16})^{8}+4\right)-(0+0)\right] - \left[\left((4)^2+4\right)-(0+0)\right]$$

$$= (24+4) - [16+4]$$

$$= 28-20 = 8$$

(10) The eq of curves are

$y^2 = 4ax \longrightarrow (1)$ $\quad y = 2\sqrt{a}\sqrt{x}$

Put $x = 0$
$y = 0$

Put $x = a$
$y^2 = 4a^2$
$y = \pm 2a$

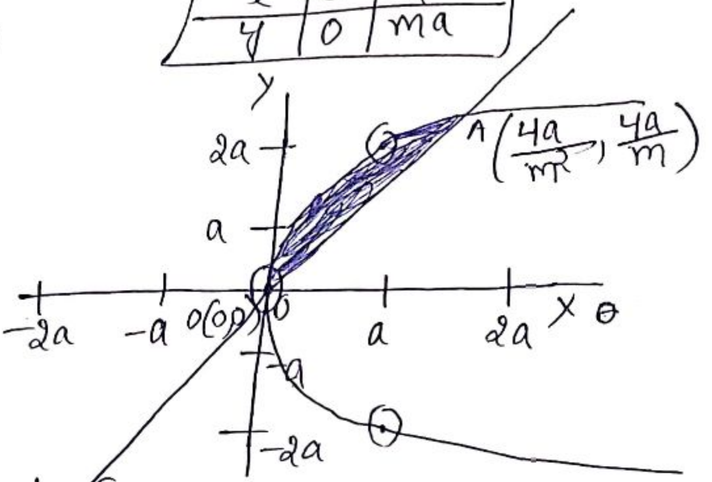
x	0	a
y	0	$\pm 2a$

$y = mx \longrightarrow (2)$

Put $x = 0$
$y = 0$

Put $x = a$
$y = ma$

x	0	a
y	0	ma



Solve eq (1) and (2)

$(mx)^2 = 4ax$

$m^2x^2 = 4ax$

$m^2x^2 - 4ax \Rightarrow x(m^2x - 4a) = 0 \Rightarrow x = 0$ or $m^2x - 4a = 0$

$x = 0 \quad x = \frac{4a}{m^2}$

when $x = 0$
For eq (2)
$y = 0$

when $x = \frac{4a}{m^2}$
then $y = m\left(\frac{4a}{m^2}\right) = \frac{4a}{m}$

Hence pt of intersections are $(0,0)\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$

Area of shaded region $= \int_0^{\frac{4a}{m^2}} y\,dx$ for parabola $- \int_0^{\frac{4a}{m^2}} y\,dx$ for line

$$= \int_0^{\frac{4a}{m^2}} 2\sqrt{a}\sqrt{x}\,dx - \int_0^{\frac{4a}{m^2}} mx\,dx$$

$$= 2\sqrt{a} \times 2\frac{x^{3/2}}{3}\Big]_0^{\frac{4a}{m^2}} - m\frac{x^2}{2}\Big]_0^{\frac{4a}{m^2}}$$

$$= \frac{4\sqrt{a}}{3}\left[\left(\frac{4a}{m^2}\right)^{3/2} - (0)^{3/2}\right] - \frac{m}{2}\left[\left(\frac{4a}{m^2}\right)^2 - (0)^2\right]$$

$$= \frac{4\sqrt{a}}{3}\left[\frac{(4)^{3/2}(a)^{3/2}}{(m^2)^{3/2}}\right] - \frac{m}{2}\left[\frac{16a^2}{m^4}\right]$$

$$= \frac{4\sqrt{a}}{3}\left[\frac{(2^2)^{3/2}(a)^{3/2}}{m^3}\right] - \frac{8a^2}{m^3}$$

$$= \frac{32}{3}\frac{a^2}{m^3} - \frac{8a^2}{m^3} = \frac{8a^2}{m^3}\left[\frac{4}{3}-1\right] = \frac{8a^2}{m^3}\left[\frac{4-3}{3}\right]$$

$$= \frac{8a^2}{3m^3}$$

---

(11) The eq of curves are

$$\frac{x^2}{9}+\frac{y^2}{4}=1 \rightarrow (1)$$

$$\frac{y^2}{4} = 1-\frac{x^2}{9}$$

$$\frac{y^2}{4} = \frac{9-x^2}{9}$$

$$y^2 = \frac{4}{9}(9-x^2)$$

$$y = \frac{2}{3}\sqrt{3^2-x^2}$$

Put $x=0$

$y = \frac{2}{\cancel{3}} \times \cancel{3} = 2$

Put $x=3$

$y=0$

x	0	3
y	2	0

$$\frac{x}{3}+\frac{y}{2}=1$$

$$\frac{y}{2} = 1-\frac{x}{3}$$

$$\frac{y}{2} = \frac{3-x}{3}$$

$$y = \frac{2}{3}(3-x)$$

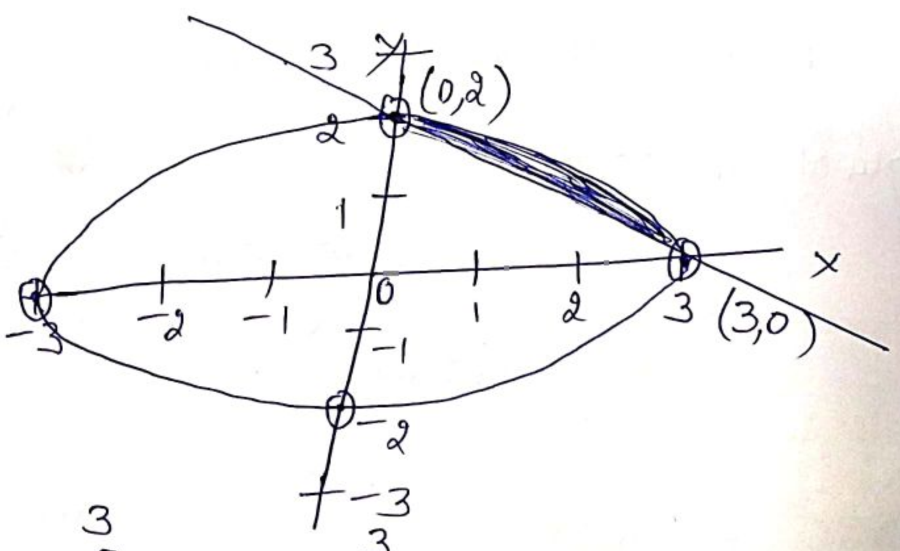
Put $x=0$

$y=2$

Put $x=3$

$y=0$

x	0	3
y	2	0



$$\text{Area of shaded region} = \int_0^3 y\,dx_{\text{for ellipse}} - \int_0^3 y\,dx_{\text{for line}}$$

$$= \int_0^3 \frac{2}{3}\sqrt{3^2-x^2}\,dx - \int_0^3 \frac{2}{3}(3-x)\,dx$$

$$= \frac{2}{3}\left[\frac{x}{2}\sqrt{3^2-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3 - \frac{2}{3}\left[3x-\frac{x^2}{2}\right]_0^3$$

$$= \frac{2}{3}\left[\left(\frac{3}{2}\sqrt{3^2-3^2}+\frac{9}{2}\sin^{-1}\cancel{\frac{3}{3}}\right)-\left(0+\frac{9}{2}\sin^{-1}0\right)\right] - \frac{2}{3}\left[\left(9-\frac{9}{2}\right)-(0-0)\right]$$

$$= \frac{2}{3}\left[0+\frac{9}{2}\times\frac{\pi}{2}\right] - \frac{2}{3}\left[\frac{18-9}{2}\right]$$

$$\frac{2}{\cancel{3}}\left[\frac{\overset{3}{\cancel{9}}\pi}{\cancel{4}_2}\right] - \frac{2}{\cancel{3}}\left[\frac{\cancel{9}^3}{\cancel{2}}\right]$$

$$\frac{3\pi}{2} - 3$$

$$3\left(\frac{\pi}{2}-1\right)$$

---

(12) The eq of curves are

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{y^2}{b^2} = 1-\frac{x^2}{a^2}$$

$$y^2 = \frac{b^2}{a^2}(a^2-x^2)$$

$$y = \frac{b}{a}\sqrt{a^2-x^2}$$

Put $x=0$

$y = b$

Put $x=a$

$y=0$

x	0	a
y	b	0

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{y}{b} = 1-\frac{x}{a}$$

$$\frac{y}{b} = \frac{a-x}{a}$$

$$y = \frac{b}{a}(a-x)$$

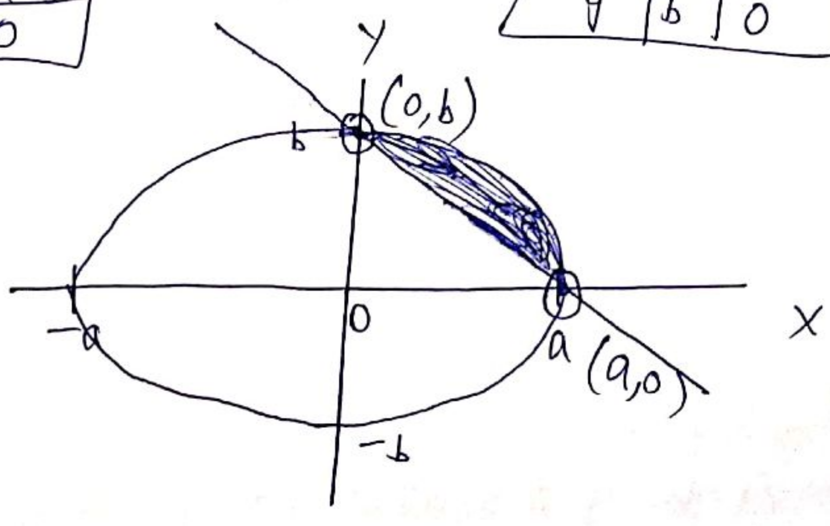
Put $x=0$

$y=b$

Put $x=a$

$y=0$

x	0	a
y	b	0



Area of shaded region $= \underset{\text{for ellipse}}{\int_0^a y\,dx} - \underset{\text{for line}}{\int_0^a y\,dx}$

$$= \frac{b}{a}\int_0^a \sqrt{a^2-x^2}\,dx - \frac{b}{a}\int_0^a (a-x)\,dx$$

$$= \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a - \frac{b}{a}\left[ax - \frac{x^2}{2}\right]_0^a$$

$$= \frac{b}{a}\left[\left(\frac{a}{2}\sqrt{a^2-a^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{a}{a}\right)\right) - \left(0 + \frac{a^2}{2}\sin^{-1}0\right)\right] - \frac{b}{a}\left[\left(a^2 - \frac{a^2}{2}\right) - (0-0)\right]$$

$$= \frac{b}{a}\left[0 + \frac{a^2}{2}\sin^{-1}(1)\right] - \frac{b}{a}\left[\frac{a^2}{2}\right]$$

$$= \frac{b}{a} \times \frac{a^2}{2} \times \frac{\pi}{2} - \frac{ab}{2}$$

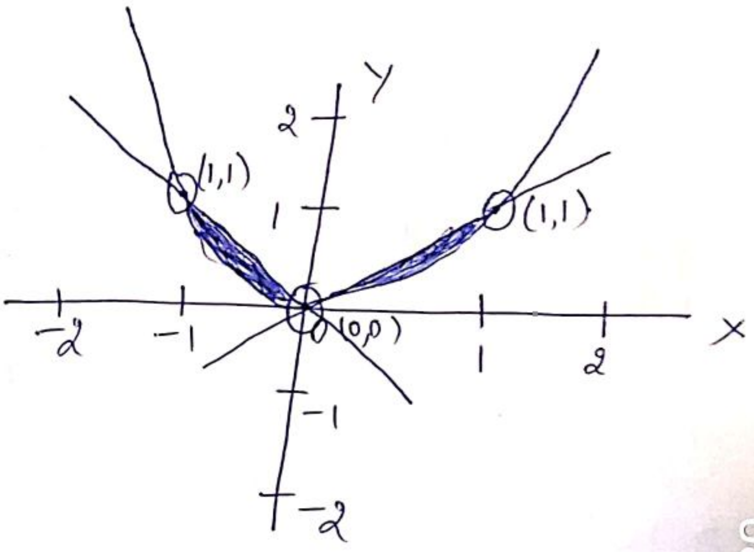
$$= \frac{ab\pi}{4} - \frac{ab}{2} = \frac{ab}{2}\left(\frac{\pi}{2} - 1\right)$$

Q→13 Let $R = \{(x,y) : y \geq x^2 \text{ and } y = |x|\}$

$R_1 = \{(x,y) : y \geq x^2\}$

$R_2 = \{(x,y) : y = |x|\}$

$R = R_1 \cap R_2$



For R_1 $\quad y = x^2$

Put $y = 0$

$x = 0$

Put $y = 1$

$x^2 = 1 \quad x = \pm 1$

x	0	± 1
y	0	1

For R_2 $\quad y = |x|$

Put $x = 0$

$y = 0$

Put $x = 1$

$y = 1$

Put $x = 2$

$y = 2$

Put $x = -1$

$y = 1$

x	0	1	2	-1
y	0	1	2	1

Hence pt of intersection are $(-1,1)$ $(0,0)$ and $(1,$

A[illegible] of shaded region $= 2\left[\int_{-1}^{1} y\,dx_{\text{for line}} - \int_{-1}^{1} y\,dx_{\text{for parabola}}\right]$

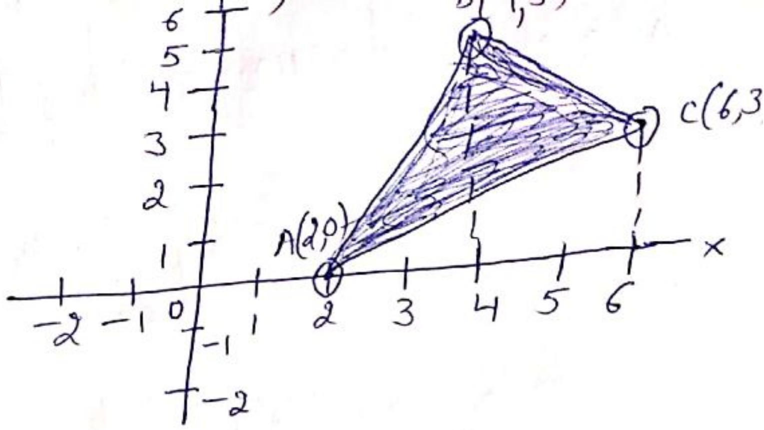
$$= 2\left[\int_{0}^{1} x\,dx - \int_{0}^{1} x^2\,dx\right] \quad (\because |x| \text{ and } x^2 \text{ are eve[n] function})$$

$$= 2\left[\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1\right]$$

$$= 2\left[\frac{1}{2}\left[1^2 - 0^2\right] - \frac{1}{3}\left[1^3 - 0^3\right]\right]$$

$$= 2\left[\frac{1}{2} - \frac{1}{3}\right] = 2\left[\frac{3-2}{6}\right] = 2\left[\frac{1}{6}\right] = \frac{1}{3}$$

14) The vertices of triangle ABC are
$A(2,0)$ $B(4,5)$ and $C(6,3)$



Equation of line AB

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-0 = \frac{5-0}{4-2}(x-2)$$

$$y = \frac{5}{2}(x-2) \longrightarrow (1)$$

Equation of line BC

$$y-5 = \frac{3-5}{6-4}(x-4)$$

$$y-5 = \frac{-\cancel{2}}{\cancel{2}}(x-4) \Rightarrow y-5 = -x+4 \Rightarrow y = 9-x \longrightarrow (2)$$

Equation of line AC

$$y-0 = \frac{3-0}{6-2}(x-2)$$

$$y = \frac{3}{4}(x-2) \longrightarrow (3)$$

$$\text{Area of shaded region} = \int_2^4 y\,dx_{\text{ for AB}} + \int_4^6 y\,dx_{\text{ for BC}} - \int_2^6 y\,dx_{\text{ for AC}}$$

$$= \int_2^4 \frac{5}{2}(x-2)dx + \int_4^6 (9-x)\,dx - \int_2^6 \frac{3}{4}(x-2)dx$$

$$= \frac{5}{2}\left(\frac{x^2}{2}-2x\right)_2^4 + \left(9x-\frac{x^2}{2}\right)_4^6 - \frac{3}{4}\left[\frac{x^2}{2}-2x\right]_2^6$$

$$= \frac{5}{2}\left[\left(\frac{16}{2}-8\right)-\left(\frac{4}{2}-4\right)\right] + \left[\left(54-\frac{36}{2}\right)-\left(36-\frac{16}{2}\right)\right] - \frac{3}{4}\left[\left(\frac{36}{2}-12\right)-\left(\frac{4}{2}-4\right)\right]$$

$$= \frac{5}{2}\left[(8-8)-(2-4)\right] + \left[(54-18)-(36-8)\right] - \frac{3}{4}\left[(18-12)-(2-4)\right]$$

$$= \frac{5}{2}\left[0+2\right] + \left[\cancel{36}-\cancel{36}+8\right] - \frac{3}{4}\left[6+2\right]$$

$$= 5+8-\frac{3}{\cancel{4}}\times\cancel{8}^{\,2}$$

$$= 5+8-2\times3$$

$$= 5+8-6 = 7$$

(15) The given lines are

$2x+y=4 \rightarrow (1)$ $\quad \cancel{x=\frac{4-y}{2}}$ $\quad y=4-2x$

$3x-2y=6 \rightarrow (2)$ $\quad y=\frac{3x-6}{2}$

$x-3y=-5 \rightarrow (3)$ $\quad y=\frac{x+5}{3}$

Solve eq (1)×2 and (2)

$4x+2y=8$
$3x-2y=6$
$7x=14$
$x=2$

Putting x in eq (1)

$4+y=4$
$y=0$

Hence pt $A(2,0)$

Solve eq (2) and (3)×3

$3x-2y=6$
$3x-9y=-15$
$-\quad +\quad +$
$7y=21$
$y=3$

Putting y in eq (2)

$3x-6=6$
$3x=12$
$x=4$

Hence pt $B(4,3)$

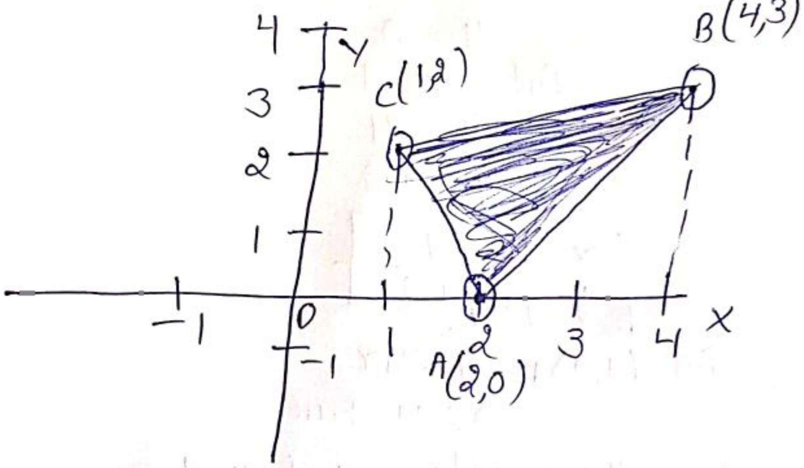
Solve eq (3)×2 and (1)

$2x+y=4$
$2x-6y=-10$
$-\quad +\quad +$
$7y=14$
$y=2$

Putting y in eq (3)

$x-6=-5$
$x=1$

Hence pt $C(1,2)$



Area of shaded region $= \int_1^4 y\,dx$ for line CB $- \int_1^2 y\,dx$ for line AC $- \int_2^4 y\,dx$ for line AB

$$= \int_1^4 \frac{x+5}{3}dx - \int_1^2 (4-2x)dx - \int_2^4 \frac{3x-6}{2}dx$$

$$= \frac{1}{3}\left[\frac{x^2}{2}+5x\right]_1^4 - \left[4x-\frac{2x^2}{2}\right]_1^2 - \frac{1}{2}\left[\frac{3x^2}{2}-6x\right]_2^4$$

$$= \frac{1}{3}\left[\left(\frac{16}{2}+20\right)-\left(\frac{1}{2}+5\right)\right] - \left[(8-4)-(4-1)\right] - \frac{1}{2}\left[\left(\frac{3}{2}\times16-24\right)-\left(\frac{3}{2}\times4-12\right)\right]$$

$$= \frac{1}{3}\left[28-\frac{11}{2}\right] - \left[4-3\right] - \frac{1}{2}\left[(24-24)-(6-12)\right]$$

$$= \frac{1}{3}\left[\frac{56-11}{2}\right] - 1 - \frac{1}{2}\left[0+6\right]$$

$$= \frac{1}{3}\left[\frac{\overset{15}{\cancel{45}}}{2}\right] - 1 - \frac{1}{2}\left(\overset{3}{\cancel{6}}\right)$$

$$\frac{15}{2} - 4 = \frac{15-8}{2} = \frac{7}{2}$$

---

(16) The eq of curve

$\{(x,y) : y^2 \leq 4x,\ 4x^2 + 4y^2 \leq 9\}$

Let $R_1 = \{(x,y) : y^2 \leq 4x\}$

$R_2 = \{(x,y) : 4x^2 + 4y^2 \leq 9\}$

$R = R_1 \cap R_2$

For R_1

$y^2 = 4x \longrightarrow (1)$

$y = 2\sqrt{x}$

Put $x = 0$

$y = 0$

Put $x = 1$

$y^2 = 4$

$y = \pm 2$

x	0	1
y	0	± 2

Put $(1,0)$ in $y^2 \leq 4x$

$0 \leq 4$ True

R_1 is the inside area of parabola

For R_2

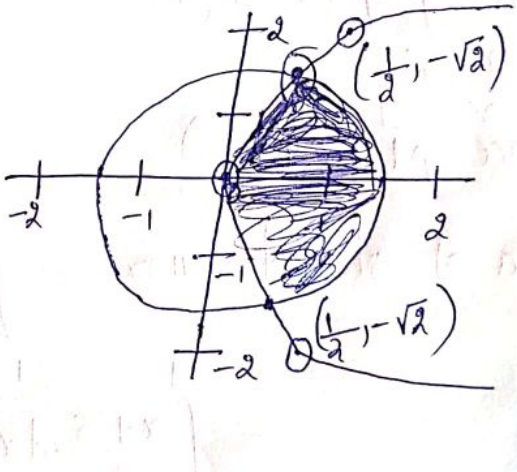
$4x^2 + 4y^2 = 9 \longrightarrow (2)$

$x^2 + y^2 = \frac{9}{4}$

$y^2 = \frac{9}{4} - x^2$

$y = \sqrt{\left(\frac{3}{2}\right)^2 - x^2}$

having radius $\frac{3}{2}$ and centre $(0,0)$



solve eq (1) and (2)

$4x^2 + 4(4x) - 9 = 0$

$4x^2 + 16x - 9 = 0$

$4x^2 + 18x - 2x - 9 = 0$

$2x(2x+9) - 1(2x+9) = 0$

$(2x-1)(2x+9) = 0$

$x = \frac{1}{2},\ -\frac{9}{2}$

when $x = \frac{1}{2}$

then eq (1)

$y^2 = \overset{2}{\cancel{4}}\left(\frac{1}{\cancel{2}}\right)$

$y = \pm\sqrt{2}$

when $x = -\frac{9}{2}$

then eq (1)

$y^2 = \overset{2}{\cancel{4}}\left(\frac{-9}{\cancel{2}}\right)$

$y^2 = -18$

$y = \sqrt{-18}$ is neglected

The pt of intersection are $\left(\frac{1}{2}, -\sqrt{2}\right)$ and $\left(\frac{1}{2}, \sqrt{2}\right)$

Area of shaded region $= 2\left[\int_0^{\frac{1}{2}} y\,dx_{\text{ for parabola}} + \int_{\frac{1}{2}}^{3/2} y\,dx_{\text{ for circle}}\right]$

$$= 2\left[\int_0^{\frac{1}{2}} 2\sqrt{x}\,dx + \int_{\frac{1}{2}}^{3/2} \sqrt{\left(\frac{3}{2}\right)^2 - x^2}\,dx\right]$$

$$= 2\left[\left[2\,\frac{x^{3/2}}{3/2}\right]_0^{\frac{1}{2}} + \left[\frac{x}{2}\sqrt{\left(\frac{3}{2}\right)^2 - x^2} + \frac{9/4}{2}\sin^{-1}\frac{x}{3/2}\right]_{\frac{1}{2}}^{3/2}\right]$$

$$= 2\left[\frac{4}{3}\left\{\left(\frac{1}{2}\right)^{3/2} - (0)^{3/2}\right\} + \left\{\left(\frac{3}{4}\sqrt{\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2} + \frac{9}{8}\sin^{-1}1\right) - \left(\frac{1}{4}\sqrt{\frac{9}{4} - \frac{1}{4}} + \frac{9}{8}\sin^{-1}\left(\frac{1}{3}\right)\right)\right\}\right]$$

$$= 2\left[\frac{4^2}{3\times\cancel{2}\sqrt{2}} + \left(0 + \frac{9}{8}\times\frac{\pi}{2} - \frac{1}{4}\sqrt{\frac{\cancel{8}\,2}{\cancel{4}}} - \frac{9}{8}\sin^{-1}\frac{1}{3}\right)\right]$$

$$= 2\left[\frac{2\sqrt{2}}{3\sqrt{2}\times\sqrt{2}} + \frac{9}{8}\times\frac{\pi}{2} - \frac{\sqrt{2}}{4} - \frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= 2\left[\left(\frac{\cancel{2}\sqrt{2}}{\cancel{6}\,3} - \frac{\sqrt{2}}{4}\right) + \frac{9}{8}\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right)\right] \qquad \boxed{\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}}$$

$$= 2\left[\sqrt{2}\left(\frac{1}{3} - \frac{1}{4}\right) + \frac{9}{8}\cos^{-1}\frac{1}{3}\right]$$

$$= 2\left[\sqrt{2}\left(\frac{4-3}{12}\right) + \frac{9}{8}\sin^{-1}\left(\sqrt{1 - \left(\frac{1}{3}\right)^2}\right)\right] \qquad \boxed{\cos^{-1}x = \sin^{-1}\sqrt{1-x^2}}$$

$$= 2\left[\frac{\sqrt{2}}{12} + \frac{9}{8}\sin^{-1}\sqrt{\frac{8}{9}}\right]$$

$$= 2\left[\frac{\sqrt{2}}{12} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3}\right]$$

$$= \frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$$

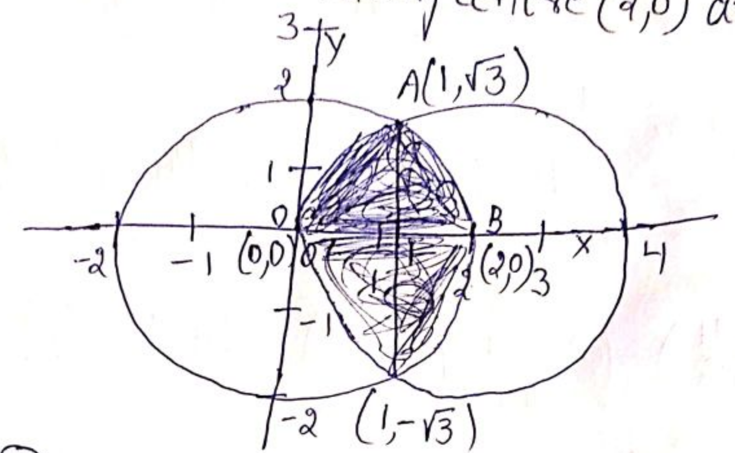
17) The equation of curves are

$$x^2+y^2=4 \rightarrow (1)$$
$$y^2 = 4-x^2$$
$$y=\sqrt{2^2-x^2}$$

having centre (0,0) radius 2

$$(x-2)^2+y^2=4 \rightarrow (2)$$
$$y^2 = 4-(x-2)^2$$
$$y=\sqrt{(2)^2-(x-2)^2}$$

having centre (2,0) and radius 2



solve eq (1) and (2)

$$x^2+y^2=(x-2)^2+y^2$$
$$x^2 = x^2+4-4x$$
$$4x=4$$
$$x=1$$

Putting x in eq (1)

$$(1)^2+y^2=4$$
$$y^2=3$$
$$y=\pm\sqrt{3}$$

pt of intersection are $(1,\sqrt{3})$ and $(1,-\sqrt{3})$

Area of shaded region

$$=2\left[\int_0^1 y\,dx_{\text{for 2nd circle}} + \int_1^2 y\,dx_{\text{for 1st circle}}\right]$$

$$=2\left[\int_0^1 \sqrt{(2)^2-(x-2)^2}\,dx + \int_1^2 \sqrt{2^2-x^2}\,dx\right]$$

$$=2\left[\left[\frac{x-2}{2}\sqrt{2^2-(x-2)^2}+\frac{4}{2}\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1 + \left[\frac{x}{2}\sqrt{2^2-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_1^2\right]$$

$$=2\left[\left\{\frac{1-2}{2}\sqrt{2^2-(1-2)^2}+2\sin^{-1}\left(\frac{-1}{2}\right)\right\}-\left\{\frac{-2}{2}\sqrt{2^2-2^2}+2\sin^{-1}\left(\frac{-2}{2}\right)\right\}+\left\{(0+2\sin^{-1}1)-\left(\frac{1}{2}\sqrt{4-1}+2\sin^{-1}\frac{1}{2}\right)\right\}\right]$$

$$2\left[\frac{-1}{2}\sqrt{4-1}+2\left(\frac{-\pi}{6}\right)-0-2\sin^{-1}(-1)+2\left(\frac{\pi}{2}\right)-\frac{1}{2}\sqrt{3}-2\left(\frac{\pi}{6}\right)\right]$$

$$2\left[\frac{-\sqrt{3}}{2}-\frac{\pi}{3}+2\left(\frac{\pi}{2}\right)+\pi-\frac{\sqrt{3}}{2}-\frac{\pi}{3}\right]$$

$$2\left[2\pi-\frac{2\pi}{3}-\frac{2\sqrt{3}}{2}\right]$$

$$2\left[\frac{6\pi-2\pi}{3}-\sqrt{3}\right]$$

$$2\left[\frac{4\pi}{3}-\sqrt{3}\right]$$

$$\frac{8\pi}{3}-2\sqrt{3}$$

---

(18) The equations of two parabola

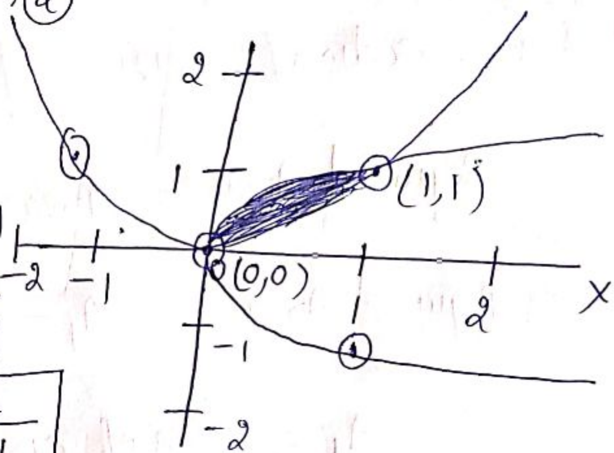
$y=x^2 \rightarrow (1)$ $\qquad$ $x=y^2 \rightarrow (2)$

Put $y=0$
$x=0$
Put $y=1$
$x^2=1$
$x=\pm 1$

x	0	± 1
y	0	1

$y=\sqrt{x}$
Put $x=0$
$y=0$
Put $x=1$
$y^2=1$
$y=\pm 1$

x	0	1
y	0	± 1



Solve eq (1) and (2)

$x=x^4$

$x-x^3=0 \Rightarrow -x(x^2-1)=0 \quad x=0 \quad x^2=1 \quad x=\pm 1$

when $x=0$ then eq (1) $y=0$ $\qquad$ when $x=1$ then eq (1) $y=(\pm 1)^2$, $y=1$

then pt of intersection $(0,0)$ and $(1,1)$

Area of shaded region $= \int_0^1 y\,dx$ for 2nd parabola $- \int_0^1 y\,dx$ for 1st parabola

$$= \int_0^1 \sqrt{x}\,dx - \int_0^1 x^2\,dx \Rightarrow \left.\frac{2x^{\frac{3}{2}}}{3}\right]_0^1 - \left.\frac{x^3}{3}\right]_0^1$$

$$\Rightarrow \frac{2}{3}\left[1^{\frac{3}{2}}-0^{3/2}\right]-\frac{1}{3}\left[1^3-0^3\right]=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$$

(19) The eq of curves are

$x^2+y^2=16 \rightarrow ①$

$y^2 = 16-x^2$

$y=\sqrt{(4)^2-x^2}$

having centre (0,0) and radius 4

$y^2 = 6x \rightarrow ②$ $\quad y=\sqrt{6x}$

Put $x=0$

$y=0$

Put $x=6$

$y^2=36$

$y=\pm 6$

x	0	6
y	0	± 6

Solve eq ① and ②

$x^2+6x-16=0$

$x^2+8x-2x-16=0$

$x(x+8)-2(x+8)=0$

$(x-2)(x+8)=0$

$x=2 \quad x=-8$

it is reject

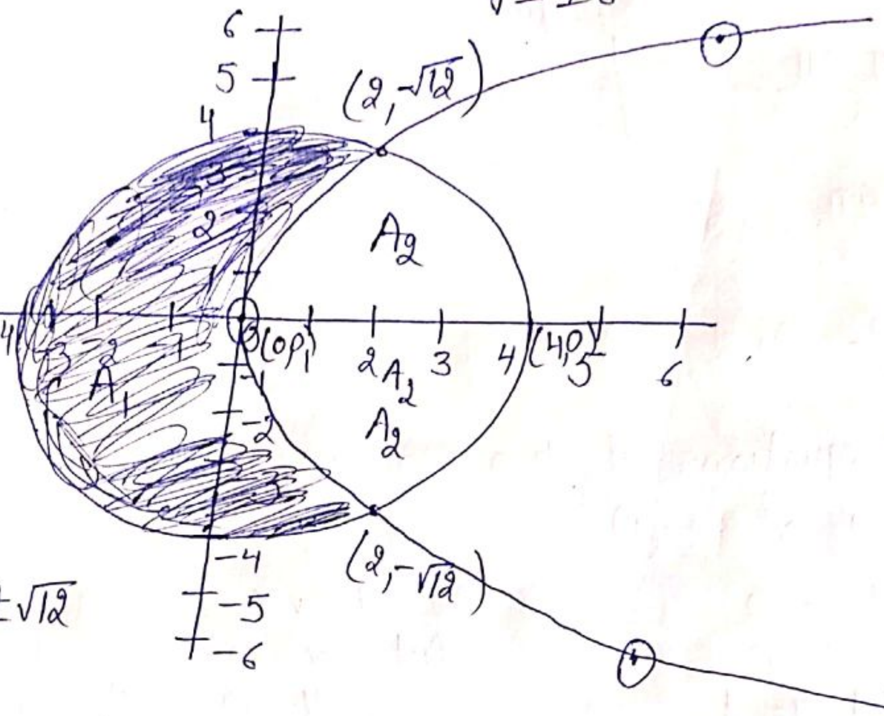
when $x=2$

then eq ②

$y^2=12$

$y=\pm\sqrt{12}$

pt of intersection $(2,\pm\sqrt{12})$



Required Area = Area of circle − Area of remaining part of circle and parabola

Required Area = $A_1 - A_2$

A_1 = Area of circle

$= \pi r^2 \qquad r=4$

$A_1 = \pi(4)^2$

$A_1 = 16\pi$

$$A_2 = 2\left[\int_0^2 y\,dx_{\text{ for parabola}} + \int_2^4 y\,dx_{\text{ for circle}}\right]$$

$$A_2 = 2\left[\int_0^2 \sqrt{6x}\,dx + \int_2^4 \sqrt{(4)^2-x^2}\,dx\right]$$

$$= 2\left[\left[\sqrt{6}\,\frac{x^{3/2}}{3/2}\right]_0^2 + \left[\frac{x}{2}\sqrt{(4)^2-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4\right]$$

$$= 2\left[\frac{2\sqrt{6}}{3}\left\{(2)^{\frac{3}{2}}-(0)^{\frac{3}{2}}\right\} + \left\{\left[0+8\sin^{-1}(1)\right] - \left[1\sqrt{16-4}+8\sin^{-1}\frac{1}{2}\right]\right\}\right]$$

$$=2\left[\frac{2\sqrt{6}}{3}\times 2\sqrt{2} + 8\times\frac{\pi}{2} - \sqrt{12} - 8\times\frac{\pi}{6}\right]$$

$$=2\left[\frac{4\sqrt{12}}{3} + 4\pi - \sqrt{12} - \frac{\cancel{8}^{4}\pi}{\cancel{6}\,3}\right]$$

$$=2\left[\left(\frac{4}{3}\sqrt{12} - \sqrt{12}\right) + \left(4\pi - \frac{4\pi}{3}\right)\right]$$

$$=2\left[\sqrt{12}\left(\frac{4-3}{3}\right) + \left(\frac{12\pi - 4\pi}{3}\right)\right]$$

$$=2\left[\frac{1}{3}\sqrt{12} + \frac{8\pi}{3}\right]$$

$$A_2 = \frac{2}{3}\sqrt{12} + \frac{16\pi}{3}$$

Required Area $= A_1 - A_2$

$$= 16\pi - \left[\frac{2}{3}\sqrt{12} + \frac{16\pi}{3}\right]$$

$$= 16\pi - \frac{2}{3}\sqrt{12} - \frac{16\pi}{3}$$

$$= \left(16\pi - \frac{16\pi}{3}\right) - \frac{2\sqrt{12}}{3}$$

$$= \left(\frac{48\pi - 16\pi}{3}\right) - \frac{2\sqrt{12}}{3}$$

$$= \frac{32\pi}{3} - \frac{2\sqrt{12}}{3}$$

$$= \frac{32\pi}{3} - \frac{4\sqrt{3}}{3}$$

$$= \frac{4}{3}\left[8\pi - \sqrt{3}\right]$$

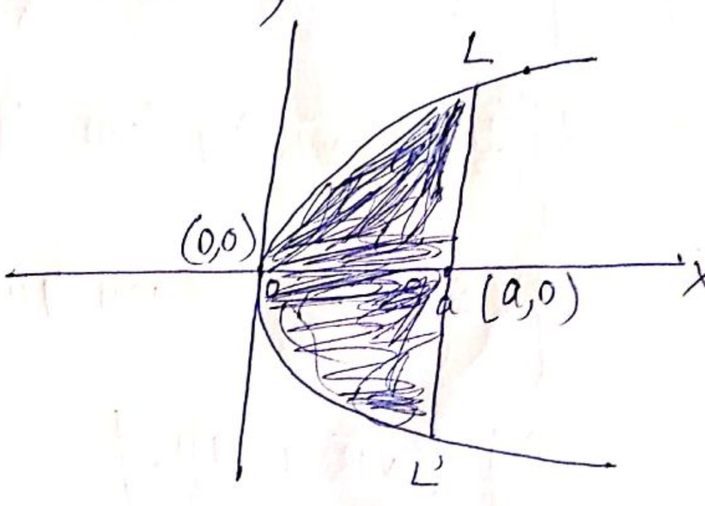
(20) The equation of parabola

$y^2 = 4ax$ vertex is at origin $(0,0)$

$y = 2\sqrt{a}\sqrt{x}$

The equation of Latus rectum $x = a$

Parabola is symmetric about x-axis



So

$$\text{Required Area} = 2\int_0^a 2\sqrt{a}\sqrt{x}\,dx$$

$$= 4\sqrt{a}\int_0^a x^{\frac{1}{2}}\,dx$$

$$= 4\sqrt{a}\times 2\,\frac{x^{\frac{3}{2}}}{3}\Big]_0^a$$

$$= \frac{8}{3}\sqrt{a}\,(a)^{3/2}$$

$$= \frac{8}{3}\sqrt{a}\times a\sqrt{a}$$

$$= \frac{8}{3}a\times a = \frac{8}{3}a^2$$