Chapter - 6

Applications of Derivatives

Sol:1 — Let side of cube $= a$

$\therefore$ vol of cube $(V) = a^3$

Diff w.r.t t.

$$\frac{dV}{dt} = 3a^2\frac{da}{dt} \quad —(1)$$

given that $\frac{dV}{dt} = +8\,cm^3/s$

(1) becomes $8 = 3a^2\frac{da}{dt}$

$$\Rightarrow \frac{da}{dt} = \frac{8}{3a^2} \quad —(2)$$

Now surface area of cube $A = 6a^2$

$$\Rightarrow \frac{dA}{dt} = 12a\frac{da}{dt} = 12a \times \frac{8}{3a^2} \quad [\text{using}$$

$$= \frac{32}{a}$$

$$\therefore \left.\frac{dA}{dt}\right]_{a=12cm} = \frac{32}{12} = +\frac{8}{3}\,cm^2/s$$

Hence rate of increase of surface area of cube is $\frac{8}{3}\,cm^2/s$.

Sol: 2 :- Let r and A be the radius and area of the circle respectively

Given $\frac{dr}{dt} = +3 cm/s$ — (1)

Now $A = \pi r^2$

$\Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi r(3)$ [By (1)]

$= 6\pi r$

$\Rightarrow \left.\frac{dA}{dt}\right]_{r=10} = 6\pi \times 10 = 60\pi$

Sol: 3 :- Let a is the edge and V be the volume of the cube.

Given $\frac{da}{dt} = +3 cm/s$ — (1)

Now $V = a^3$

$\Rightarrow \frac{dV}{dt} = 3a^2 \frac{da}{dt} = 3a^2(3)$ — (By (1))

$\Rightarrow \left.\frac{dV}{dt}\right]_{a=10cm} = 9(10)^2 = 900$

$\therefore$ Vol. of cube is increasing at the rate $900 cm^3/s$.

Sol: 4 :- Given $\frac{dr}{dt} = +0.7 cm/s$ — (1)

r be the radius of the circle

Circumference of the circle, $C = 2\pi r$

$\Rightarrow \frac{dC}{dt} = 2\pi \frac{dr}{dt} = 2\pi(0.7)$ [Using (1)]

$= 1.4\pi$

$\therefore$ Rate of increase of circumference of the circle is $1.4\pi cm/s$.

Sol: 5:- (i) $y(x) = x^2+2x-5$

$\Rightarrow y'(x) = 2x+2 = 2(x+1)$

$y(x) = 0 \Rightarrow x = -1$

Intervals are $(-\infty,-1)$ and $(-1,\infty)$

when $x \in (-\infty,-1)$ then $y'(x) < 0$

when $x \in (-1,\infty)$, then $y'(x) > 0$

$\therefore$ $y(x)$ is strictly increasing in $(-1,\infty)$ and strictly decreasing in $(-\infty,-1)$.

(ii)

$y(x) = 10-6x-2x^2$

$y'(x) = -6-4x$

$= -2(2x+3)$

$y'(x) = 0 \Rightarrow x = -\frac{3}{2}$

$\therefore$ Intervals are $(-\infty,-\frac{3}{2})$ and $(-\frac{3}{2},\infty)$, when

$x \in (-\infty,-\frac{3}{2})$ then $y'(x) = -2(-) > 0$

when $x \in (-\frac{3}{2},\infty)$ then $y'(x) = -2(+) < 0$

$\therefore$ $y(x)$ is strictly increasing in $(-\infty,-\frac{3}{2})$ and strictly decreasing in $(-\frac{3}{2},\infty)$.

(iii) $y(x) = -2x^3-9x^2-12x+1$

$y'(x) = -6x^2-18x-12$

$= -6(x^2+3x+2)$

$= -6(x+1)(x+2)$

$y'(x) = 0 \Rightarrow x = -2,-1$

Intervals are $(-\infty,-2),(-2,-1),(-1,\infty)$

when $x \in (-\infty,-2)$, $y'(x) = -6(-)(-) < 0$

when $x \in (-2,-1)$, $y'(x) = -6(-)(+) > 0$

when $x \in (-1,\infty)$, $y'(x) = (-6)(+)(+) < 0$

$\therefore$ $y(x)$ is strictly increasing in $(-2,-1)$ and strictly decreasing in $(-\infty,-2) \cup (-1,\infty)$

(iii)

$$f(x) = 6 - 9x - x^2$$

$$\Rightarrow f'(x) = -9 - 2x = -(2x+9)$$

$$f'(x) = 0 \Rightarrow x = -\frac{9}{2}$$

Intervals are : $(-\infty, -\frac{9}{2})$ and $(-\frac{9}{2}, \infty)$

When $x \in (-\infty, -\frac{9}{2})$, $f'(x) = (-)(-) > 0$

When $x \in (-\frac{9}{2}, \infty)$, $f'(x) = (-)(+) < 0$

$\therefore$ $f(x)$ is strictly increasing in $(-\infty, -\frac{9}{2})$ and strictly decreasing in $(-\frac{9}{2}, \infty)$.

Sol: 6 :- Let $y = f(x) = \log(1+x) - \frac{2x}{2+x}$

$$f'(x) = \left[\frac{1}{1+x} - \frac{(2+x)2 - 2x \times 1}{(2+x)^2}\right]$$

$$= \frac{1}{1+x} - \frac{4}{(2+x)^2} = \frac{(2+x)^2 - 4 - 4x}{(1+x)(2+x)^2}$$

$$= \frac{4 + x^2 + 4x - 4 - 4x}{(1+x)(2+x)^2} = \frac{x^2}{(x+1)(x+2)^2}$$

$\Rightarrow f'(x) > 0$ for $x > -1$

$\therefore$ $f(x)$ is strictly increasing function of x throught its domain

---

Sol: 7 :- $y = x^3 - x + 1$ —(1)

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 1$$

$\therefore$ Slope of tangent to (1) is

$$\left.\frac{dy}{dx}\right]_{x=2} = 3(2)^2 - 1 = 11.$$

Sol: 8 :- Given curve is $x = a\cos^3\theta$, $y = a\sin^3\theta$

$$\Rightarrow \frac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta) \text{ and}$$

$$\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{3a\sin^2\theta\cos\theta}{3a\cos^2\theta(-\sin\theta)}$$

$\left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{4}} = -\tan\frac{\pi}{4} = -1$ $\therefore$ Slope of normal $= -\frac{1}{m} = 1$

Scanned by CamScanner

Sol: 9. — Given curve is $y = x^2 - 2x + 7$ — (1)

(a) Parallel to line $2x - y + 9 = 0$

Let $P(h,k)$ be a point on (1)

$\therefore\ k = h^2 - 2h + 7$ — (2)

Diff (1) w.r.t x

$$\frac{dy}{dx} = 2x - 2$$

m_1 = Slope of tangent to (1) $= \left.\frac{dy}{dx}\right]_{P(h,k)}$

$= 2h - 2$

m_2 = Slope of given line

$2x - y + 9 = 0$

$= -\frac{2}{-1} = 2$

Acc. to Question $m_1 = m_2 \Rightarrow 2h - 2 = 2 \Rightarrow h = 2$

& (2) $\Rightarrow k = (2)^2 - 2(2) + 7 = 7$

$\therefore\ P(h,k) = (2,7)$

Now equation of tangent to (1) at P is

$y - 7 = m(x-2)$

$\Rightarrow y - 7 = 2(x-2)$

$\Rightarrow 2x - y + 3 = 0$

(b) Perpendicular to the line $5y - 15x = 13$

Let given curve is $y = x^2 - 2x + 7$ — (1)

Let $P(h,k)$ be a point on (1)

$\therefore\ k = h^2 - 2h + 7$ — (2)

(1) $\Rightarrow \frac{dy}{dx} = 2x - 2$

m_1 = Slope of tangent to (1) $= \left.\frac{dy}{dx}\right]_{P(h,k)}$

$= 2h - 2$

m_2 = Slope of given line

$15y - 15x = 13 = \frac{15}{5} = 3$

Acc. to Question $m_1 m_2 = -1$

$\Rightarrow (2h-2)3 = -1 \Rightarrow h = \frac{5}{6}$

$(2) \Rightarrow \quad k = \frac{217}{36}$

$\therefore \quad P = \left(\frac{5}{6}, \frac{217}{36}\right)$

$\therefore$ Equation of tangent to (1) is

$$y - \frac{217}{36} = m_1\left(x - \frac{5}{6}\right)$$

$$\Rightarrow y - \frac{217}{36} = (2h-2)\left(x - \frac{5}{6}\right) = \left(2\times\frac{5}{6} - 2\right)\left(x - \frac{5}{6}\right)$$

$$\Rightarrow \frac{36y - 217}{36} = \frac{-2(6x-5)}{36}$$

$$\Rightarrow 36y - 217 = -12x + 10$$

$$\Rightarrow 12x + 36y = 227$$

(ii) Given curve is

$$y = x^4 - 6x^3 + 13x^2 - 10x + 5 \qquad -(1)$$

$$\Rightarrow \frac{dy}{dx} = 4x^3 - 18x^2 + 16x - 10$$

$m =$ slope of tangent to $(1) = \left.\frac{dy}{dx}\right|_{(0,5)} = -10$

Eq^n of tangent to (1) at (0,5) is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 5 = -10(x - 0)$$

$\Rightarrow 10x + y = 5$ and eq^n of normal to (1)

at (0,5) is $\quad [\because m_1 m_2 = -1]$

$$y - 5 = -\frac{1}{m}(x - 0)$$

$$y - 5 = \frac{1}{10}(x)$$

$$\Rightarrow 10y - 50 = x$$

$$\Rightarrow x - 10y + 50 = 0$$

(iv) Given curve is $y = x^3$

$$\Rightarrow \frac{dy}{dx} = 3x^2$$

$m =$ Slope of tangent to (1)

$$= \left.\frac{dy}{dx}\right|_{(1,1)}$$

$= 3(1)^2 = 3$

$\therefore$ Eqn of tangent to (1) at (1,1) is

$y-1 = 3(x-1) \Rightarrow 3x-y=2$

And equation of normal to (1) at (1,1)

is $y-1 = -\frac{1}{3}(x-1)$

$\Rightarrow 3y-3 = -x+1$

$\Rightarrow x+3y=4$

Sol: 10. (i) $\sqrt{25.3} = \sqrt{25+0.3}$

Let $x = 25$ and $\Delta x = 0.3$

Taking $y = \sqrt{x}$ — (1)

$\Rightarrow y + \Delta y = \sqrt{x+\Delta x}$ — (2)

(2) − (1) $\Rightarrow \Delta y = \sqrt{x+\Delta x} - \sqrt{x}$ — (3)

using approximation $\Delta y \approx dy$

(3) $\Rightarrow dy = \sqrt{25+0.3} - \sqrt{25}$

$\Rightarrow \frac{dy}{dx} \times \Delta x = \sqrt{25.3} - 5$ [By def. $dy = \frac{dy}{dx} \cdot \Delta x$]

$\Rightarrow \sqrt{25.3} = 5 + \frac{dy}{dx} \cdot \Delta x = 5 + \frac{1}{2\sqrt{x}}(0.3)$

$= 5 + \frac{1}{2\sqrt{25}}(0.3) = 5 + \frac{0.3}{10} = 5 + 0.03$

$\therefore \sqrt{25.3} \simeq 5.03$

(ii) $\sqrt{49.5} = \sqrt{49+0.5}$

Let $x = 49$, $\Delta x = 0.5$

Taking $y = \sqrt{x}$ — (1)

$\Rightarrow y + \Delta y = \sqrt{x+\Delta x}$ — (2)

(2) − (1) $\Rightarrow \Delta y = \sqrt{x+\Delta x} - \sqrt{x}$ — (3)

using approximation $\Delta y \simeq dy$

(3) $\Rightarrow dy = \sqrt{49+0.5} - \sqrt{49}$

$\Rightarrow \frac{dy}{dx} \times \Delta x = \sqrt{49.5} - 7$

$\Rightarrow \sqrt{49.5} = 7 + \frac{dy}{dx} \times \Delta x = 7 + \frac{1}{2\sqrt{x}}(0.5)$

$= 7 + \frac{1}{2\sqrt{49}} \times (0.5) = 7 + \frac{0.5}{14} = \frac{98.5}{14}$

$= 7.0357$

$\therefore \sqrt{49.5} \approx 7.036$

(ii) $(0.009)^{1/3} = \sqrt[3]{0.008 + 0.001}$

Let $x = 0.008$, $\Delta x = 0.001$

taking $y = \sqrt[3]{x}$ — (1)

$\Rightarrow y + \Delta y = \sqrt[3]{x + \Delta x}$ — (2)

(2) − (1) $\Rightarrow \Delta y = \sqrt[3]{x + \Delta x} - \sqrt[3]{x}$ — (3)

using approximation $\Delta y \simeq dy$

(3) $\Rightarrow dy = \sqrt[3]{0.009} - \sqrt[3]{0.008}$

$\Rightarrow \frac{dy}{dx} \Delta x = \sqrt[3]{0.009} - 0.2$

$\Rightarrow \sqrt[3]{0.009} = 0.2 + \frac{dy}{dx} \Delta x = 0.2 + \frac{1}{3x^{2/3}} (0.001)$

$= 0.2 + \frac{0.001}{3(0.008)^{2/3}} = 0.2 + \frac{0.001}{3 \times 0.04}$

$= \frac{2}{10} + \frac{1}{120} = \frac{25}{120} = 0.208$

$\therefore \sqrt[3]{0.009} \simeq 0.208$

(iv) $(255)^{\frac{1}{4}} = \sqrt[4]{256 - 1}$

Let $x = 256$, $\Delta x = -1$

taking $y = \sqrt[4]{x}$ — (1)

$\Rightarrow y + \Delta y = \sqrt[4]{x + \Delta x}$ — (2)

(2) − (1) $\Rightarrow \Delta y = \sqrt[4]{x + \Delta x} - \sqrt[4]{x}$ — (3)

using approximation, $\Delta y \simeq dy$

(3) $\Rightarrow dy = \sqrt[4]{255} - \sqrt[4]{256}$

$\Rightarrow \frac{dy}{dx} \cdot \Delta x = \sqrt[4]{255} - 4$

$\Rightarrow \sqrt[4]{255} = 4 + \frac{dy}{dx} \cdot \Delta x = 4 + \frac{1}{4x^{3/4}} (-1)$

$= 4 - \frac{1}{4(256)^{3/4}} = 4 - \frac{1}{4 \times 64}$

$= \frac{1023}{256} = 3.996$

$\therefore \sqrt[4]{255} \simeq 3.996$

Scanned by CamScanner

(v)

$\sqrt{0.0037} = \sqrt{0.0036 + 0.0001}$

Let $x = 0.0036$, $\Delta x = 0.0001$

Taking $y = \sqrt{x}$ — (1)

$\Rightarrow y + \Delta y = \sqrt{x + \Delta x}$ — (2)

(2) − (1) $\Rightarrow \Delta y = \sqrt{x+\Delta x} - \sqrt{x}$ — (3)

using approximation $\Delta y \simeq dy$

(3) $\Rightarrow dy = \sqrt{0.0037} - \sqrt{0.0036}$

$\Rightarrow \frac{dy}{dx} \cdot \Delta x = \sqrt{0.0037} - 0.06$

$\Rightarrow \sqrt{0.0037} = 0.06 + \frac{dy}{dx} \cdot \Delta x$

$= 0.06 + \frac{1}{2\sqrt{x}}(0.0001)$

$= 0.06 + \frac{0.0001}{2\sqrt{0.0036}} = 0.06 + \frac{0.0001}{0.12}$

$= \frac{6}{100} + \frac{1}{1200} = \frac{73}{1200} = \frac{0.73}{12}$

$= 0.0608$

$\therefore \sqrt{0.0037} \simeq 0.061$

(vi) $(26.57)^{\frac{1}{3}} = \sqrt[3]{27 - 0.43}$

Let $x = 27$, $\Delta x = -0.43$

Taking $y = \sqrt[3]{x}$ — (1)

$\Rightarrow y + \Delta y = \sqrt[3]{x + \Delta x}$ — (2)

(2) − (1) $\Rightarrow \Delta y = \sqrt[3]{x+\Delta x} - \sqrt[3]{x}$ — (3)

using approximation $\Delta y \simeq dy$

(3) $\Rightarrow dy = \sqrt[3]{26.57} - \sqrt[3]{27}$

$\Rightarrow \frac{dy}{dx} \cdot \Delta x = \sqrt[3]{26.57} - 3$

$\Rightarrow \sqrt[3]{26.57} = 3 + \frac{dy}{dx} \cdot \Delta x = 3 + \frac{1}{3x^{2/3}}(-0.43)$

$= 3 + \frac{1}{3(27)^{2/3}}(-0.43)$

$$= 3 - \frac{0.43}{3\times 9}$$

$$= 3 - \frac{43}{2700} = \frac{8057}{2700} = 2.984$$

$$\therefore \sqrt[3]{26.57} \simeq 2.984.$$

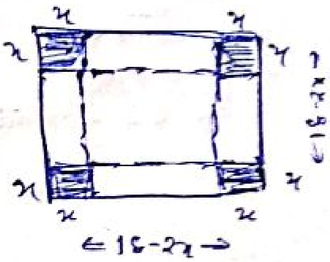
Sol: 11:- Let x cm be the side of the square to be cut from each corner of the given square piece of tin of side 18 cm

$\therefore$ Therefore the box.

$l = 18 - 2x$ cm

$b = 18 - 2x$ cm

and $h = x$ cm



$\therefore$ Volume of the box

$$V = lbh = (18-2x)^2 x$$

$$= (324 + 4x^2 - 72x)x$$

$$\Rightarrow V = 324x + 4x^3 - 72x^2$$

$$\Rightarrow \frac{dv}{dx} = 324 + 12x^2 - \cancel{144x}$$

$$\frac{dv}{dx} = 0 \Rightarrow 12x^2 - 144x + 324 = 0$$

$$\Rightarrow 12(x^2 - 12x + 27) = 0$$

$$\Rightarrow (x-3)(x-9) = 0$$

$$\Rightarrow x = 3, 9$$

but $x = 9 \Rightarrow l = b = 0$ which is impossible

$$\therefore x = 3$$

Now $\frac{d^2y}{dx^2} = 24x - 144$

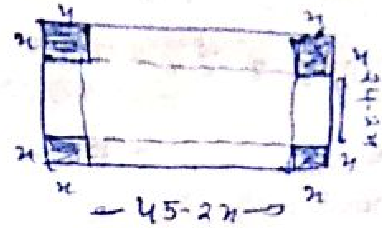
$$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=3} = 72 - 144 < 0$$

$\Rightarrow$ V is maximum when $x = 3$

$\therefore$ Hence Vol of the box will be maximum when the side of the square to be cut from each corner $= 3$ cm.

Sol: 12:- Let x cm. be the side of the square to be cut

$\therefore$ For the box



$l = (45-2x)$ cm

$b = (24-2x)$ cm

and $h = x$ cm

$\therefore$ Vol. $V = l \times b \times h$

$= (45-2x)(24-2x) \cdot x$

$= 4x^3 - 138x^2 + 1080x$

$= 2(2x^3 - 69x^2 + 540x]$

$\Rightarrow \frac{dV}{dx} = 2[6x^2 - 138x + 540]$

$= 12[x^2 - 23x + 90]$

$\frac{dV}{dx} \Rightarrow (x-5)(x-18) = 0$

$\Rightarrow x = 5$ as $x \neq 18$

Now $\frac{d^2V}{dx^2} = 12(2x - 23]$

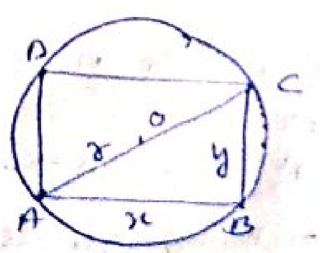
$\Rightarrow \left.\frac{d^2V}{dx^2}\right]_{x=5} = 12[10-23] < 0$

$\therefore V$ is maximum when $x = 5$.

Hence volume of box will be maximum when side of the square to be cut $= 5$ cm

---

Sol: 13:- Let $x \times y$ be the rectangle inscribed in the circle of diameter d



$\Rightarrow$ In $\triangle ABC$

$x^2 + y^2 = d^2$ — (1)

Now Area of the rectangle ABCD

$A = xy$

$\Rightarrow A^2 = x^2y^2$

Let $Z = A^2$

$\therefore \quad z = x^2(d^2 - x^2)$ -- [using (1)

$= d^2x^2 - x^4$

$\Rightarrow \frac{dz}{dx} = 2d^2x - 4x^3$

$\Rightarrow \frac{dz}{dx} = 0 \Rightarrow 2d^2x - 4x^3 = 0$

$\Rightarrow d^2 = 2x^2$

$\therefore x = \frac{d}{\sqrt{2}}$

Now $\frac{d^2z}{dx^2} = 2d^2 - 12x^2$

$\Rightarrow \left.\frac{d^2z}{dx^2}\right|_{x=\frac{d}{\sqrt{2}}} = 2d^2 - 12 \times \frac{d^2}{2} < 0$

$\therefore$ z or area of the rectangle is maximum

when $x = \frac{d}{\sqrt{2}}$

(1) $\Rightarrow \frac{d^2}{2} + y^2 = d^2$

$\Rightarrow y^2 = \frac{d^2}{2}$

$\therefore y = \frac{d}{\sqrt{2}}$

$\Rightarrow x = y$

Hence the rectangle of maximum area is a square

Sol: 14.- Let r be the radius and h be the height of the cylinder

Given surface area of the cylinder

$(A) = 2\pi rh + 2\pi r^2$ - (1)

Vol. of cylinder

$V = \pi r^2 h$

$= \pi r^2 \left[\frac{A - 2\pi r^2}{2\pi r}\right]$

$\Rightarrow V = \frac{1}{2}[rA - 2\pi r^3]$

$\Rightarrow \frac{dV}{dr} = \frac{1}{2}(A - 6\pi r^2)$

$\Rightarrow \frac{dV}{dr} = 0 \Rightarrow A = 6\pi r^2$

$\Rightarrow 2\pi rh + 2\pi r^2 = 6\pi r^2$

$\Rightarrow 2\pi rh = 4\pi r^2$

$r = h/2$

Now $\frac{d^2V}{dr^2} = \frac{1}{2}(0 - 12\pi r) = -6\pi r$

$\Rightarrow \left.\frac{d^2V}{dr^2}\right]_{r=\frac{h}{2}} < 0$

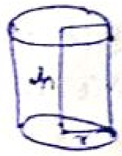
$\therefore$ V is maximum when

$r = \frac{h}{2}$ or $h = 2r$

$\therefore$ Vol. of the cylinder of given surface is maximum provided its height is equal to diameter of its base

---

Sol:15:- Let r cm be the radius and h cm be the height of the can



the surface area of the can i.e $A = 2\pi rh + 2\pi r^2$ —(1)

and volume of the can

$V = \pi r^2 h = 100 cm^3$ —(2)

From (1) and (2)

$A = 2\pi r \times \frac{100}{\pi r^2} + 2\pi r^2$

$= \frac{200}{r} + 2\pi r^2$

$\Rightarrow \frac{dA}{dr} = -\frac{200}{r^2} + 4\pi r$

$\frac{dA}{dr} = 0 \Rightarrow 4\pi r = \frac{200}{r^2}$

$\Rightarrow \pi r^3 = 50$

$\Rightarrow r^3 = 50/\pi$

Now $\dfrac{d^2A}{dr^2} = \dfrac{400}{r^3} + 4\pi$

$\Rightarrow \left.\dfrac{d^2A}{dr^2}\right|_{r^3=\frac{50}{\pi}} > 0$

$\therefore$ A is minimum when $r^3 = \dfrac{50}{\pi}$

$r = \sqrt[3]{\dfrac{50}{\pi}}$

(2) $\Rightarrow h = \dfrac{100}{\pi r^2} = \dfrac{100}{\pi\left(\frac{50}{\pi}\right)^{2/3}} = \dfrac{100\,\pi^{2/3}}{(50)^{2/3}\pi}$

$= \dfrac{50\times 2}{(50)^{2/3}}\pi^{1/3}$

$= \dfrac{2\times 50^{1/3}}{\pi^{1/3}} = 2\left(\dfrac{50}{\pi}\right)^{1/3}$

$\therefore$ Dimensions of the can having maximum surface area $r = \left(\dfrac{50}{\pi}\right)^{1/3}$ cm and

$h = 2\left(\dfrac{50}{\pi}\right)^{1/3}$ cm

Sol: 16:- Let AB = 28 m be the length of the wire

AC = x m and

CB = (28-x) m be the

two pieces of the wire

A ——— C ——— B (x, $28-x$)

Let the piece AC is to be made into square of side a meters and CB into a circle of radius r metres.

$\therefore$ Perimeter of the square $= x$

$\Rightarrow 4a = x$

$\Rightarrow a = \dfrac{x}{4}$

and circumference of the circle $= 28-x$

$\Rightarrow 2\pi r = 28-x$

$\Rightarrow r = \dfrac{28-x}{2\pi}$

Combined area of the square and the circle

$A = a^2 + \pi r^2 = \dfrac{x^2}{16} + \pi\dfrac{(28-x)^2}{4\pi^2}$

$\Rightarrow A = \dfrac{x^2}{16} + \dfrac{(28-x)^2}{4\pi}$

$$\Rightarrow \frac{dA}{dx} = \frac{2x}{16} + \frac{2(28-x)}{4\pi}(-1) = \frac{x}{8} - \frac{(28-x)}{2\pi}$$

$$\frac{dA}{dx} = 0 \Rightarrow \frac{x}{8} = \frac{28-x}{2\pi}$$

$$\Rightarrow \pi x = 112 - 4x$$

$$\Rightarrow x(4+\pi) = 112$$

$$x = \frac{112}{4+\pi}$$

Now $\frac{d^2A}{dx^2} = \frac{1}{8} + \frac{1}{2\pi}$

$$\Rightarrow \left.\frac{d^2A}{dx^2}\right|_{x=\frac{112}{4+\pi}} > 0$$

$\Rightarrow$ A is minimum when

$$x = \frac{112}{4+\pi}$$

$\therefore AC = \frac{112}{4+\pi}$ and

$$CB = 28 - \frac{112}{4+\pi} = \frac{112 + 28\pi - 112}{4+\pi}$$

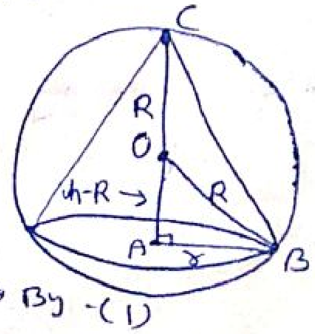
$$= \frac{28\pi}{4+\pi}$$

Hence the combined area will be minimum if length of the pieces are $\frac{112}{4+\pi}$ and $\frac{28\pi}{4+\pi}$ m.

Sol:17:- Let r be the radius and h be the height of the cone. In fig $OA = h - R$

In ΔOAB,

$$R^2 = r^2 + (h-R)^2 \quad \text{— (1)}$$



Vol of cone

$$V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3} h\left[R^2 - (h-R)^2\right]$$

$\rightarrow$ By -(1)

$$= \frac{\pi}{3} h\left[R^2 - h^2 - R^2 + 2hR\right] = \frac{\pi}{3} h(2hR - h^2)$$

$$= \frac{\pi}{3}(2Rh^2 - h^3) \quad \text{— (2)}$$

$$\Rightarrow \frac{dv}{dh} = \frac{\pi}{3}(4Rh - 3h^2)$$

$$\frac{dv}{dh} = 0 \Rightarrow \frac{\pi}{3}(4Rh - 3h^2) = 0$$

$$\Rightarrow 4Rh = 3h^2$$

$$\Rightarrow 4R = 3h$$

$$\Rightarrow h = 4R/3$$

Now $\frac{d^2v}{dh^2} = \frac{\pi}{3}(4R - 6h)$

$$\Rightarrow \left.\frac{d^2v}{dh^2}\right]_{h=\frac{4R}{3}} = \frac{\pi}{3}\left(4R - 6 \times \frac{4R}{3}\right)$$

$$= \frac{\pi}{3}(4R - 8R) < 0$$

$\therefore$ V is maximum when $h = \frac{2R}{3}$

$$\frac{\text{Vol. of cone}}{\text{Vol. of Sphere}} = \frac{\frac{\pi}{3}(2Rh^2 - h^3)}{\frac{4}{3}\pi R^3}$$

$$= \frac{\left[2R\left(\frac{4R}{3}\right)^2 - \left(\frac{4R}{3}\right)^3\right]}{4R^3}$$

$$= \frac{2 \times \frac{16R^3}{9} - \frac{64R^3}{27}}{4R^3}$$

$$= \cancel{96} \; \frac{96 - 64}{4 \times 27} = \frac{32}{4 \times 27}$$

$$= \frac{8}{27}$$

Hence volume of largest con $= \frac{8}{27}$ volume of the sphere hence the result.

Sol: 18 :- Let r, h, l be the radius, height and slant height of the cone.

Volume of cone $(V) = \frac{1}{3}\pi r^2 h$ (Given) — (i)

C.S.A. of Cone $(A) = \pi r l$.

$\Rightarrow A^2 = \pi^2 r^2 l^2$

$= \pi^2 r^2 \left(r^2 + \frac{9V^2}{\pi^2 r^4} \right)$ — [by (i)]

$= \pi^2 r^2 \frac{(\pi^2 r^6 + 9V^2)}{\pi^2 r^4}$

Let $Z = A^2 = \pi^2 r^4 + \frac{9V^2}{r^2}$

$\Rightarrow \frac{dZ}{dr} = 4\pi^2 r^3 - \frac{18V^2}{r^3}$

$\frac{dZ}{dr} = 0 \Rightarrow 4\pi^2 r^3 = \frac{18V^2}{r^3}$

$\Rightarrow 2\pi^2 r^6 = 9\left(\frac{1}{3}\pi r^2 h\right)^2$

$\Rightarrow 2\pi^2 r^6 = 9 \times \frac{1}{9} \pi^2 r^4 h^2$

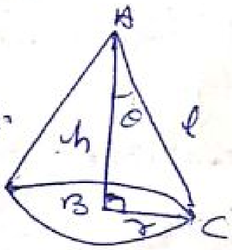
$\Rightarrow 2r^2 = h^2 \Rightarrow r = \frac{h}{\sqrt{2}}$

Now $\frac{d^2Z}{dr^2} = 12\pi^2 r^2 + \frac{54V^2}{r^4}$

$\Rightarrow \left.\frac{d^2Z}{dr^2}\right]_{r=\frac{h}{\sqrt{2}}} > 0$

$\Rightarrow$ C.S.A. (Z) of the cone is minimum when $r = \frac{h}{\sqrt{2}}$. Hence C.S.A of the cone will be minimum if $h = \sqrt{2} r$.

Sol: 19:- Let θ be the semi-vertical angle of the cone of maximum volume and of given slant height.



Vol. of cone $V = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi (l^2 - h^2) h$$

$$= \frac{1}{3}\pi (l^2 h - h^3)$$

$\Rightarrow \frac{dV}{dh} = \frac{1}{3}\pi (l^2 - 3h^2)$ $\frac{dV}{dh} = 0 \Rightarrow l^2 = 3h^2$

$\Rightarrow \frac{l}{h} = \sqrt{3}$

$\Rightarrow$ In ΔABC, $\sec\theta = \sqrt{3}$

$\Rightarrow \tan\theta = \sqrt{\sec^2\theta - 1} = \sqrt{3-1}$

$\tan\theta = \sqrt{2}$

Now $\frac{d^2V}{dh^2} = \frac{1}{3}(\pi (0 - 6h)$

$$\left.\frac{d^2V}{dh^2}\right|_{l^2 = 3h^2} < 0$$

$\Rightarrow$ V is maximum when $\theta = \sqrt{2}$

$\Rightarrow \theta = \tan^{-1}\sqrt{2}$

Hence semi-vertical angle of cone of maximum Vol. is $\tan^{-1}(\sqrt{2})$.

Sol: 20:- Let θ be the semi-vertical angle of the cone with radius r, height h, slant height l,

Given: Surface area of cone

$$A = \pi r l + \pi r^2 \quad -(1)$$

Vol. of cone $= V = \frac{1}{3}\pi r^2 h$

$\Rightarrow V^2 = \frac{1}{9}\pi^2 r^4 h^2$

$$= \frac{1}{9}\pi^2 r^4 (l^2 - r^2)$$

$= \frac{1}{9}\pi^2 r^4 \left[\frac{A^2 + \pi^2 r^4 - 2\pi A r^2 - \pi^2 r^4}{\pi^2 r^2} \right]$ [using (1)]

$\cancel{= \frac{1}{9}\pi^2 r^4 [A^2 +}$

$= \frac{1}{9} r^2 [A^2 - 2\pi A r^2]$

$= \frac{1}{9}[A^2 r^2 - 2\pi A r^4]$

Let $z = V^2 = \frac{A}{9}[A r^2 - 2\pi r^4]$

$\Rightarrow \frac{dz}{dr} = \frac{A}{9}[2Ar - 8\pi r^3]$

$\frac{dz}{dr} = 0 \Rightarrow \frac{A}{9}(2Ar - 8\pi r^3) = 0$

$\Rightarrow 2Ar = 8\pi r^3$

$\Rightarrow A = 4\pi r^2$

$\Rightarrow \pi r l + \pi r^2 = 4\pi r^2$ — [By (1)]

$\Rightarrow \pi r l = 3\pi r^2$

$\Rightarrow l = 3r$

$\Rightarrow \frac{r}{l} = \frac{1}{3}$

$\Rightarrow$ In $\triangle ABC$: $\sin\theta = \frac{1}{3}$

$\Rightarrow \theta = \sin^{-1}\left[\frac{1}{3}\right]$

Now $\frac{d^2 z}{dr^2} = \frac{A}{9}[2A - 2\pi r^2]$

$\Rightarrow \left.\frac{d^2 z}{dr^2}\right|_{r=\frac{1}{3}} < 0$

$\Rightarrow$ z is maximum when $\theta = \sin^{-1}\left[\frac{1}{3}\right]$

Scanned by CamScanner

M.C.Q. : chapter 2

Sol 1 :- (A) ie $\frac{4}{5}$

Sol : 2 → (C) ie $\frac{15}{17}$

Sol : 3 → (A) ie $-\frac{\pi}{2}$

Sol : 4 → (B) ie $\frac{5\pi}{6}$

Sol : 5 :- (C) ie $\frac{2\pi}{3}$

Sol : 6 :- (B) ie $-\frac{\pi}{3}$

Sol : 7 – (D) ie $\frac{\pi}{6}$

Sol : 8 – (D) ie 1

Chapter - 3
matrices

Sol :- 1 (B) ie $\frac{\pi}{3}$

Sol : 2 :- (C) ie $m = n$

Sol : 3 :- (D) ie 512

Sol : 4 :- (A) skew symmetric matrix.

Sol . 5 (B) – A null matrix

Sol . 6 (D) . None of these

Sol . 7 . (B) I

Chapter - 4.
Determinants

Sol : 1 . $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $f(x) = x^2 - 2x - 3$, then

$$f(A) = A^2 - 2A - 3 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3$$

Sol : 2 (C) ie $|A|^{n-1}$

Sol : 3 . (C) ie $|A|^{n-1}$

Sol : 4 . (D) ie $\frac{1}{\det A}$

Sol : 5 :- (A) ie $(AB)^{-1} = B^{-1}A^{-1}$

Sol : 6 :- (D) ie $k^3 |A|$

Sol : 7 :- (C) ie singular.

Chapter - 5
Continuity and Differentiability

Sol 1 :- (A) ie $-\frac{\sin\sqrt{x}}{2\sqrt{x}}$

Sol : 2 : (A) ie 1.

Sol : 3 :- (A) ie $\frac{1}{1+x^2}$

Sol : 4 - (A) ie $\frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$

Sol: 5 :- (C) i.e. $a^x \log a$

Sol: 6 - (B) = $-\operatorname{cosec}^2 x$

Sol: 7 (C) = $\frac{a}{ax+b}$

Sol: 8 : (A) = $-ne^{-nx}$

Sol: 9 (A) = 11

Sol: 10 (D) None of these

Chapter - 6

Applications of Derivatives

Sol: 1 :- (A) = $60\pi \, cm^2/sec$

Sol: 2 :- (B) = $-\frac{1}{3}$

Sol: 3 :- (B) = 764

Sol: 4 :- (A) = (0, 2)

Sol: 5 (A) = $1.4\pi \, cm/sec$.

Scanned by CamScanner