Chapter - 4

# Determinants

Sol. 1 :- Let vertices of The triangle be A(4, 2), B (4, 5) and C(-2, 2)

Area of triangle ABC = Absolute Value of $\frac{1}{2}\begin{vmatrix} 4 & 2 & 1 \\ 4 & 5 & 1 \\ -2 & 2 & 1 \end{vmatrix}$

$= \frac{1}{2} | 4(5-2) - 2(4+2) + 1(8+10)]$

$= 9$ sq. units

Thus area of required triangle is 9 sq. units

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Sol. 2 :- Let Vertices of the triangle be A(1, -1), B(2, 4) and C(-3, 5).

Area of triangle ABC = Absolute value of $\frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & 4 & 1 \\ -3 & 5 & 1 \end{vmatrix}$

$= \frac{1}{2}\left| 1(4-5)+1(2+3)+1(10+12) \right|$

$= \frac{1}{2}\left|-1+5+22\right| = \frac{1}{2}\left|26\right|$

$= 13$ sq. units

thus area of required triangle is 13 sq. units

Q3:- Let
(i)

$$\Delta = \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}$$

operating $C_1 \to C_1 + C_2$, we get

$$\Delta = \begin{vmatrix} x+a & a & x+a \\ y+b & b & y+b \\ z+c & c & z+c \end{vmatrix} = 0$$

[$\because$ $C_1$ and $C_3$ are Identical]

(ii)

$$\Delta = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

operate $R_1 \to R_1 + R_2 + R_3$, we get

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

[$\because$ all the elements of row $R_1$ are zero]

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(x) Let $A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

We write $A = AI$

$$\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Applying $C_1 \to C_1 + 2C_2$.

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

Applying $C_2 \to C_2 + C_1$ :

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = A\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$$

Applying $C_2 \to \frac{1}{2} C_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

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(xi) Let $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

We write $A = IA$

$$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & -4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - R_1$ , we get

$$\begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 + 2R_2$ :

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -1 & 2 \end{bmatrix} A$$

Applying $R_2 \to \frac{1}{2} R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -1/2 & 1 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

Sol 4:-

$$\Delta = \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & bc & ab+ac \\ 1 & ca & bc+ba \\ 1 & ab & ca+cb \end{vmatrix}$$

operating $C_3 \to C_3 \to C_2$ we get

$$\Delta = \begin{vmatrix} 1 & bc & ab+bc+ca \\ 1 & ca & bc+ba+ca \\ 1 & ab & ca+cb+ab \end{vmatrix}$$

Taking $ab+bc+ca$ common from $C_3$:

$$\Delta = (ab+bc+ca) \begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix}$$

$$= (ab+bc+ca)(0) = 0$$

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Sol. 5:- Let $\Delta = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$

Taking $a, b, c$ as common from $R_1$, $R_2$ and $R_3$:

$$\Delta = abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

Again taking $a, b, c$ common from $C_1$, $C_2$ & $C_3$

$$\Delta = a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

Sol.6 :- Let $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$.

operating $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$

Taking $(b-a)$ and $(c-a)$ as common from $R_2$ and $R_3$.

$$\Delta = (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+c \\ 0 & 1 & c+a \end{vmatrix}$$

Expand by $C_1$

$$\Delta = (b-a)(c-a)\begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix}$$

$$\Delta = (b-a)(c-a)[(c+a)-(b+a)]$$

$$= (b-a)(c-a)(c-b)$$

$$\Delta = (a-b)(b-c)(c-a)$$

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Sol.7 :- Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

operate $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$.

$$\Delta = \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & b^3-a^3 & c^3-a^3 \end{vmatrix}$$

Taking $(b-a)$ and $(c-a)$ as common from $C_2$ and $C_3$.

$$\Delta = (b-a)(c-a)\begin{vmatrix} 1 & 1 \\ b^2+ab+a^2 & c^2+ac+a^2 \end{vmatrix}$$

$$= (b-a)(c-a)[c^2+ac+a^2-b^2-ab-a^2]$$
$$= (b-a)(c-a)(c^2-b^2+ac-ab)$$
$$= (b-a)(c-a)((c-b)(c+b)+a(c-b))$$
$$= (b-a)(c-a)(c-b)(c+b+a)$$
$$= (a-b)(b-c)(c-a)(a+b+c)$$

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Sol. 8:- Let $\Delta = \begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}$

multiply $R_1$, $R_2$ and $R_3$ by $x, y$ & $z$.

$$\Delta = \frac{1}{xyz}\begin{vmatrix} x^2 & x^3 & xyz \\ y^2 & y^3 & xyz \\ z^2 & z^3 & xyz \end{vmatrix}$$

Taking $xyz$ as common from $C_3$

$$\Delta = \frac{xyz}{xyz}\begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}$$

operating $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} x^2 & x^3 & 1 \\ y^2-x^2 & y^3-x^3 & 0 \\ z^2-x^2 & z^3-x^3 & 0 \end{vmatrix}$$

Taking $(y-x)$ & $(z-x)$ as common from $R_2$ & $R_3$.

$$\Delta = (y-x)(z-x)\begin{vmatrix} x^2 & x^3 & 1 \\ y+x & y^2+xy+x^2 & 0 \\ z+x & z^2+zx+x^2 & 0 \end{vmatrix}$$

Expand along $C_3$.

$$\Delta = (y-x)(z-x)\begin{vmatrix} y+x & y^2+xy+x^2 \\ z+x & z^2+zx+x^2 \end{vmatrix}$$

$$\therefore \Delta = (y-x)(z-x)[(y+x)(z^2+zx+x^2)-(z+x)(y^2+xy+x^2)]$$

$$= (y-x)(z-x)[yz^2+xyz+yx^2+xz^2+x^3-zy^2-xyz-zx^2-xy^2-yx^2-x^3]$$

$$= (y-x)(z-x)[yz^2+xyz+xz^2-xy^2-zy^2-xyz]$$

$$= (y-x)(z-x)[z(yz+xy+xz)-y(xy+yz+zx)]$$

$$= (y-x)(z-x)(z-y)(xy+yz+zx)$$

$$= (x-y)(y-z)(z-x)(xy+yz+zx)$$

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Sol. 9:- Let $\Delta = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

operate $C_1 \to C_1+C_2+C_3$:

$$\Delta = \begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix}$$

Taking $5x+4$ common from $C_1$

$$\Delta = (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix}$$

operate $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\Delta = (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & 4-x & 0 \\ 0 & 0 & 4-x \end{vmatrix}$$

Taking $4-x$ as common from $R_2$ and $R_3$:

$$\Delta = (5x+4)(4-x)(4-x)\begin{vmatrix} 1 & 2x & 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expand along $C_1$, we get

$$\Delta = (5x+4)(4-x)^2\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= (5x+4)(4-x)^2(1-0)$$

$$\Delta = (5x+4)(4-x)^2$$

x —— x —— x

Sol. 16:→ Let $\Delta = \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

operate $C_1 \to C_1+C_2+C_3$, we get

$$\Delta = \begin{vmatrix} 3y+k & y & y \\ 3y+k & y+k & y \\ 3y+k & y & y+k \end{vmatrix}$$

Taking $3y+k$ as common from $C_1$:

$$\Delta = (3y+k)\begin{vmatrix} 1 & y & y \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix}$$

operate $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\Delta = (3y+k)\begin{vmatrix} 1 & y & y \\ 0 & k & 0 \\ 0 & 0 & k \end{vmatrix}$$

Expand along $C_1$:

$$\Delta = (3y+4)\, 1 . \begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix}$$

$$= (3y+k)(k^2-0)$$

$$\Delta = k^2(3y+k)$$

Sol. 11 :- ~~Let Δ = a+b+c~~

Let $\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

operate $R_1 \to R_1 + R_2 + R_3$ :

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Taking $a+b+c$ as common from $R_1$

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Expand along $C_1$

$$\Delta = (a+b+c)(a+b+c) \begin{vmatrix} 1 & 1 \\ b-c-a & 2b \end{vmatrix}$$

$$\Delta = (a+b+c)^2 [(2b - (b-c-a)]$$

$$\Delta = (a+b+c)^3$$

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Sol: 12 :- Let $\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

operate $C_1 \to C_1 - bC_3$, $C_2 \to C_2 + aC_3$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix}$$

Taking $1+a^2+b^2$ as common from $C_1$ and $C_2$ :

$$\Delta = (1+a^2+b^2)\begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$$

Expand along $C_1$ :

$$\Delta = (1+a^2+b^2)^2\left[1\begin{vmatrix} 1 & 2a \\ -a & 1-a^2-b^2 \end{vmatrix} - 0 + b\begin{vmatrix} 0 & -2b \\ 1 & 2a \end{vmatrix}\right.$$

$$= (1+a^2+b^2)^2\left[(1-a^2-b^2)+2a^2+b(0+2b)\right]$$

$$= (1+a^2+b^2)^2(1+a^2+b^2) = (1+a^2+b^2)^3$$

x ———— x

Sol: 13 :→ Let $\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$

multiply $C_1, C_2$ and $C_3$ by $a, b$ and $c$ :

$$\Delta = \frac{1}{abc}\begin{vmatrix} a(a^2+1) & ab^2 & ac^2 \\ a^2b & b(b^2+1) & bc^2 \\ a^2c & cb^2 & c(c^2+1) \end{vmatrix}$$

Taking $a, b$ and $c$ as common from $R_1, R_2$ & $R_3$.

$$\Delta = \frac{abc}{abc}\begin{vmatrix} a^2+1 & b^2 & c^2 \\ a^2 & b^2+1 & c^2 \\ a^2 & b^2 & c^2+1 \end{vmatrix}$$

operate $C_1 \to C_1 + C_2 + C_3$ :

$$\Delta = \begin{vmatrix} 1+a^2+b^2+c^2 & b^2 & c^2 \\ 1+a^2+b^2+c^2 & b^2+1 & c^2 \\ 1+a^2+b^2+c & b^2 & c^2+1 \end{vmatrix}$$

operate $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$ :

$$\Delta = (1+a^2+b^2+c^2)\begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

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Expand by $C_1$

$$\Delta = (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= (1+a^2+b^2+c^2)(1-0)$$

$$= 1+a^2+b^2+c^2$$

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Sol 14 (i) The given system of equations can be expressed as $AX=B$

where $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Now $|A| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3-4 = -1 \neq 0$

Hence, A is non singular matrix so given system of equation is Consistent.

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(ii) The given system of equations can be expressed as $AX=B$.

where $A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix} = 5(18+10)+1(12-25) + 4(-4-15)$$

$$= 140-13-76 = 51 \neq 0$$

$\therefore$ hence A is non-singular matrix so given system is consistent.

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Sol 15 :- (i) The given system of equations can be expressed as $AX=B$, where

$$A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$\therefore$ A is non-singular matrix so $A^{-1}$ exist and given system has a unique solution.

$A_{11} = (-1)^{1+1} 3 = 3$. $A_{12} = (-1)^{1+2} 7 = -7$

$a_{21} = -2$, $A_{22} = 5$

$$\text{adj } A = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}' = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{1}\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

Now. $AX = B \Rightarrow X = A^{-1}B$.

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

on equating, we get $x = 2, y = -3$.

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Sol. 15 (ii)

$$2x + y + z = 1$$
$$x - 2y - z = \frac{3}{2}$$
$$3y - 5z = 9$$

the given system of equations can be expressed as $AX = B$, where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and $B = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{vmatrix} = 2(10+3) - 1(-5-0) + 1(3-0)$$

$$= 26 + 5 + 3 = 34 \neq 0$$

∴ A is non-singular matrix so $A^{-1}$ exist and given system has a unique solution

$A_{11} = 13, \quad A_{12} = 5 \quad A_{13} = 3$

$A_{21} = 8 \quad A_{22} = -10 \quad A_{23} = -6$

$A_{31} = 1 \quad A_{32} = 3 \quad A_{33} = -5$

$$\text{adj } A = \begin{bmatrix} 13 & 5 & 3 \\ 8 & -10 & -6 \\ 1 & 3 & -5 \end{bmatrix}' = \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{34} \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

Now $AX = B \Rightarrow X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$

$$= \frac{1}{34} \begin{bmatrix} 13+12+9 \\ 5-15+27 \\ 3-9-45 \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ -3/2 \end{bmatrix}$$

hence, $x = 1$, $y = \frac{1}{2}$ and $z = -\frac{3}{2}$

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Sol. 15 (iii) :- $\begin{bmatrix} x-y+z=4 \\ 2x+y-3z=0 \\ x+y+z=2 \end{bmatrix}$

$A =$ , $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$$|A| = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} = 1(1+3) + 1(2+3) + 1(2-1)$$

$$= 4 + 5 + 1 = 10 \neq 0$$

$A^{-1}$ exist.

$A_{11} = 4$ ; $A_{12} = -5$ , $A_{13} = 1$

$A_{21} = 2$ $A_{22} = 0$ $A_{23} = -2$

$A_{31} = 2$ $A_{32} = 5$ $A_{33} = 3$

$$\text{adj } A = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

Now $AX = B \Rightarrow X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4+0+6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\Rightarrow x = 2, y = -1, z = 1$

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(iv) $A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & -1 \\ 3 & -1 & -2 \end{bmatrix}$, $X \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

$|A| = 2(4+1) - 3(-2-3) + 3(-1+6)$

$= 10 + 15 + 15 = 40 \neq 0$

$\therefore A^{-1}$ exist.

$A_{11} = 5$ ; $A_{12} = 5$ , $A_{13} = 5$

$A_{21} = 3$ $A_{22} = -13$ $A_{23} = 11$

$A_{31} = 9$ $A_{32} = 1$ ; $A_{33} = -7$

$$\text{adj } A = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

Now $AX = B \Rightarrow X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$\Rightarrow x = 1, y = 2$ and $z = -1$

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(V) $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

$$|A| = 1(12-5) + 1(9+10) + 2(-3-8)$$
$$= 7 + 19 - 22 = 4 \neq 0$$

$\therefore A^{-1}$ exist

$A_{11} = 7,\ A_{12} = -19,\ A_{13} = -11$

$A_{21} = 1,\ A_{22} = -1,\ A_{23} = -1$

$A_{31} = -3,\ A_{32} = 11,\ A_{33} = 7$

$$\text{Adj } A = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

Now $AX = B \Rightarrow X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 2, y = 1$ and $z = 3$

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Sol:16:- Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

we have $A_{11} = 3$ $A_{12} = -12$, $A_{13} = 6$

$A_{21} = 1$, $A_{22} = 5$, $A_{23} = 2$

$A_{31} = -11$, $A_{32} = -1$, $A_{33} = 5$

$$\text{adj } A = \begin{bmatrix} 3 & -12 & 6 \\ 1 & 5 & 2 \\ -11 & -1 & 5 \end{bmatrix}' = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$$

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Sol:17:- $L.H.S = \begin{vmatrix} x & x^2 & x^4 \\ y & y^2 & y^4 \\ z & z^2 & z^4 \end{vmatrix}$

Taking $x, y$ and $z$ common from $R_1, R_2$ and $R_3$.

$$= xyz\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix}$$

operate $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & x & x^3 \\ 0 & y-x & y^3-x^3 \\ 0 & z-x & z^3-x^3 \end{vmatrix}$$

Expanding by $C_1$:

$= xyz[(y-x)(z^3-x^3)-(y^3-x^3)(z-x)]$

$= xyz[(y-x)(z-x)(z^2+x^2+zx)-(y-x)(y^2+x^2+xy)(z-x)]$

$= xyz(y-x)(z-x)[z^2+x^2+zx-y^2-x^2-xy]$

$= xyz(y-x)(z-x)[(z^2-y^2)+x(z-y)]$

$= xyz(y-x)(z-x)(z-y)[z+y+z]$

$= (xyz)(x-y)(y-z)(z-x)(x+y+z)$

$= R.H.S$

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Sol 18:- $L.H.S = \begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$

operate $C_3 \to C_3 + C_1$

$$\Delta = \begin{vmatrix} \alpha & \alpha^2 & \alpha+\beta+\gamma \\ \beta & \beta^2 & \alpha+\beta+\gamma \\ \gamma & \gamma^2 & \alpha+\beta+\gamma \end{vmatrix}$$

Take $(\alpha+\beta+\gamma)$ common from $C_3$

$$\Delta = (\alpha+\beta+\gamma)\begin{vmatrix} \alpha & \alpha^2 & 1 \\ \beta & \beta^2 & 1 \\ \gamma & \gamma^2 & 1 \end{vmatrix}$$

operate $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$\Delta = (\alpha+\beta+\gamma)\begin{vmatrix} \alpha & \alpha^2 & 1 \\ \beta-\gamma & \beta^2-\alpha^2 & 0 \\ \gamma-\alpha & \gamma^2-\alpha^2 & 0 \end{vmatrix}$$

Expand by $C_3$.

$$\Delta = (\alpha+\beta+\gamma)(\beta-\alpha)(\gamma-\alpha)\begin{vmatrix} 1 & \beta+\alpha \\ 1 & \gamma+\alpha \end{vmatrix}$$

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$= (\alpha+\beta+\gamma)(\beta-\alpha)(\gamma-\alpha)(\gamma+\alpha-\beta-\alpha)$

$= (\alpha+\beta+\gamma)(\beta-\alpha)(\gamma-\alpha)(\gamma-\beta)$

$\Delta = (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$