Chapter - 3
Matrices

Sol. 1 :- A 2x2 matrix can be given as

(i)
$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2$, $a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$

$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$, $a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$

$\therefore A = \begin{bmatrix} 2 & 9/2 \\ 9/2 & 8 \end{bmatrix}$

(ii) $a_{11} = \frac{1}{1} = 1$; $a_{12} = \frac{1}{2}$; $a_{21} = \frac{2}{1} = 2$, $a_{22} = \frac{2}{2} = 1$

$\therefore A = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$

$a_{11} = \frac{(1+2)^2}{2} = \frac{9}{2}$, $a_{12} = \frac{(1+4)^2}{2} = \frac{25}{2}$

$a_{21} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$, $a_{22} = \frac{(2+4)^2}{2} = \frac{36}{2} = 18$

$A = \begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$

Sol. 2 (i) $a_{ij} = \frac{1}{2}|-3i+j|$

A. 3x4 matrix can be given as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$a_{11} = \frac{1}{2}|-3+1| = \frac{1}{2}|-2| = \frac{1}{2}(2) = 1$

$a_{12} = \frac{1}{2}|-3+2| = \frac{1}{2}|-1| = \frac{1}{2}(1) = \frac{1}{2}$

$a_{13} = \frac{1}{2}|-3+3| = \frac{1}{2}|0| = 0$, $a_{14} = \frac{1}{2}|-3+4| = \frac{1}{2}|1| = \frac{1}{2}$

Similarily $a_{21} = \frac{5}{2}$, $a_{22} = 2$, $a_{23} = \frac{3}{2}$

$a_{24} = 1$, $a_{31} = 4$, $a_{32} = \frac{7}{2}$, $a_{33} = 3$

$a_{34} = \frac{5}{2}$

$\therefore \quad A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & 1/2 \\ 5/2 & 2 & 3/2 & 1 \\ 4 & 7/2 & 3 & 5/2 \end{bmatrix}$

(ii) $a_{ij} = 2i - j$

$a_{11} = 2-1 = 1$, $a_{12} = 2-2 = 0$, $a_{13} = 2-3 = -1$, $a_{14} = 2-4 = -2$
$a_{21} = 4-1 = 3$, $a_{22} = 4-2 = 2$, $a_{23} = 4-3 = 1$, $a_{24} = 4-4 = 0$
$a_{31} = 6-1 = 5$, $a_{32} = 6-2 = 4$, $a_{33} = 6-3 = 3$, $a_{34} = 6-4 = 2$

$\therefore A \quad \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$

Sol. 3: we have $(X+Y)+(X-Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

$\Rightarrow \quad 2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$

Also $(X+Y)-(X-Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

$\Rightarrow 2Y = \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix} \Rightarrow Y = \frac{1}{2}\begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$

$\Rightarrow Y = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$

Sol 4:-(i) $X+Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ - (i)

$X-Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ - (ii)

Adding (i) & (ii)

$2X + 0 = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

Now putting $X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$ in (i), we get

$\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \Rightarrow Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

$\Rightarrow Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(ii)

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \quad -\ (i)$$

$$3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} \quad -\ (ii)$$

(i) × 2 − (ii) × 3 gives

$$4x - 9X = 2\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 3\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$-5X = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} -6 & 6 \\ 3 & -15 \end{bmatrix}$$

$$-5X = \begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix}$$

$$X = -\frac{1}{5}\begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

putting value of X in (i), we get

$$2\begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}$$

$$3Y = \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$Y = \frac{1}{3}\begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

Sol.5:- Given $2X+Y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$

$$2X+\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$$

$$2X=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}-\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$$

$$2X=\begin{bmatrix}-2 & -2\\ -4 & -2\end{bmatrix}$$

$$X=\begin{bmatrix}-1 & -1\\ -2 & -1\end{bmatrix}$$

Sol.6 :- Given $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}2 & 6\\ 0 & 2x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$

$$\begin{bmatrix}2+y & 6\\ 1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$

on comparing corresponding elements, we get

$2+y=5 \Rightarrow y=3$

and $2x+2=8 \Rightarrow 2x=6$

$\Rightarrow x=3$

Sol.7:- $A^2=A\cdot A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$

$$=\begin{bmatrix}4+0+1 & 0+0-1 & 2+0+0\\ 4+2+3 & 0+1-3 & 2+3+0\\ 2-2+0 & 0-1+0 & 1-3+0\end{bmatrix}=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix}$$

Now $A^2-5A+6I=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix}-5\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}+6\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$$=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix}+\begin{bmatrix}-10 & 0 & -5\\ -10 & -5 & -15\\ -5 & 5 & 0\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$$

$$\begin{bmatrix} 5-10+6 & -1+6+0 & 2-5+0 \\ 9-10+0 & -2-5+6 & 5-15+0 \\ 0-5+0 & -1+5+0 & -2+0+6 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

Sol. 8 :- $A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$$L.H.S = A^3 - 6A^2 + 7A + 2I$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} + \begin{bmatrix} -30 & 0 & -48 \\ -12 & -24 & -30 \\ -48 & 0 & -78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21-30+7+2 & 0+0+0+0 & 34-48+14+0 \\ 12-12+0+0 & 8-24+14+2 & 23-30+7+0 \\ 34-48+14+0 & 0+0+0+0 & 55-78+21+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$$\therefore A^3 - 6A^2 + 7A + 2I = 0$$

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Sol. 9 (i) Given $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$A'A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$\begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha\sin\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \Rightarrow A'A = I$

(ii) Given $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, $A' = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$

$A'A = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \Rightarrow A'A = I$

Sol. 10:- (i) Let $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ and $A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A')$

$= \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} \right)$

$= \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$

$\therefore P = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$ $\therefore P' = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} = P$

$\therefore P = \frac{1}{2}(A + A')$ is a symmetric matrix

Also let $Q = \frac{1}{2}(A - A')$

$= \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} \right) = \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

$Q = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$G' = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -G$

$\therefore G = \frac{1}{2}(A-A')$ is a skew symmetric matrix

Now $P+G = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A$

(ii) $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$, $A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Let $P = \frac{1}{2}[A+A'] = \frac{1}{2}\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$\therefore P' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = P$

$\therefore P = \frac{1}{2}(A+A')$ is symmetric matrix

Also let $G = \frac{1}{2}(A-A') = \frac{1}{2}\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$G' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -G$

$\therefore G = \frac{1}{2}(A-A')$ is a skew symmetric matrix

Now $P+G = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = A$

(iii) let $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ and $A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

Let $P = \frac{1}{2}(A+A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right)$

$$= \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$$

$$P' = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} = P$$

$\therefore$ $P = \frac{1}{2}(A+A')$ is a symmetric matrix

Also let $G = \frac{1}{2}(A-A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right)$

$$= \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

$$G' = \begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = -G$$

Now $P+G = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = A$$

(iv)

$A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ and $A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A')$

$= \frac{1}{2}\left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right)$

$P = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

$P' = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

$\Rightarrow P' = P$

$\therefore$ P is symmetric matrix

Also $Q = \frac{1}{2}(A - A') = \frac{1}{2}\left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right)$

$= \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

$Q' = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} = -Q$

$\Rightarrow Q' = -Q$.

Q is skew symmetric matrix

Now $P + Q = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} = A$

Thus A is represented as a sum of symmetric and skew symmetric matrix

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Sol. 11 :- (i) Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

we write $A = IA$

$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$, we get

$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 + \frac{1}{5} R_2$, we get

$\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -2 & 1 \end{bmatrix} A$

Again applying $R_2 \to \frac{1}{5}R_2$ we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

(ii) Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

We write $A = IA$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - R_1$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

(iii) $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$, we write $A = IA$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - 3R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}.$$

(iv) Let $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$.

we write $A = AI$

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Applying $C_1 \to C_1 - C_2$ we get

$$\begin{bmatrix} -1 & 3 \\ -2 & 7 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

Applying $C_1 \to (-1)C_1$, we get

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = A\begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$$

Applying $C_2 \to C_2 - 3C_1$.

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = A\begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$$

Applying $C_1 \to C_1 - 2C_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A\begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

$\therefore A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$

(v) Let $A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

we write $A = AI$

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Applying $C_1 \to C_1 - C_2$ we get

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

Applying $C_2 \to C_2 - C_1$;

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} = A\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

Applying $C_1 \to C_1 - 3C_2$;

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

$\therefore A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

(vi) Let $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

we write ~~A = AI~~ $A = IA$

$\therefore \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - R_2$, we get

$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - R_1$:

$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$

Applying $R_1 \to R_1 - 2R_2$:

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$

$\therefore A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

(vii) Let $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

we write $A = AI$

$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Applying $C_1 \to C_1 - 2C_2$, we get

$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

Applying $C_2 \to C_2 - C_1$, we get

$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$

Applying $C_1 \to C_1 - C_2$:

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

(viii) Let $A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

We write $A = IA$

$$\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$ we get

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 3R_1$:

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

(ix) Let $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

We write $A = IA$

$$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$:

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A$$

Applying $R_1 \to R_1 - 3R_2$:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

(xiii) Let $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

We write $A = IA$

$\therefore \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 + R_2$, we get

$\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 + R_1$;

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A$

Applying $R_1 \to R_1 + R_2$;

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A$

$\therefore A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$