Chapter - 1 $\boxed{\text{Solutions}}$ of Question bank.

Relations and Functions

Ans 1:- Let $x_1 = 2 > 0$ and $x_2 = 5 > 0 \in R$

$\therefore f(x_1) = f(2) = 1$

and $f(x_2) = f(5) = 1$

$\therefore f(x_1) = f(x_2) = 1$

But $x_1 \neq x_2$. Thus the function is not one-one.

Show that signum fn: $R \to R$ is neither one-one nor onto.
Let $x_1 = 1$ & $x_2 = 3$ be two real no.
$f(x_1) = f(1) = 1$; $f(x_2) = f(3) = 1$
$\therefore f(x_1) = f(x_2)$ for $x_1 \neq x_2$
$\therefore f: R \to R$ is not one-one
Also $y = 2 \in R$
But range of $f = \{-1, 0, 1\}$
$\therefore f: R \to R$ is not onto

Ans 2:- $f(x) = |x|$

Let $x_1 = 1$ and $x_2 = -1$ be two real numbers

$f(x_1) = f(1) = |1|$; $f(x_2) = f(-1) = |-1| = 1$

$\therefore f(x_1) = f(x_2)$ for $x_1 \neq x_2$

$\therefore f: R \to R$ is not one-one

Also Let $y = -1 \in R$

$\therefore f(x) = -1 \Rightarrow |x| = -1$

which is not possible as $|x|$ is always positive

$f: R \to R$ is not onto

Sol 3:- Let $n_1 = 1$ and $n_2 = 2 \in N$, the domain of f

$\therefore f(n_1) = f(1) = \frac{1+1}{2} = 1$ ($\because n_1 = 1$ is odd)

and $f(n_2) = f(2) = \frac{2}{2} = 1$ ($\because n_2$ is even)

$\therefore f(n_1) = f(n_2) = 1$

But $n_1 \neq n_2$ Thus the function is not one-one.

Sol 4:- We have $f(x) = \cos x$ and $g(x) = 3x^2$

$\therefore gof(x) = g(f(x)) = g(\cos x) = 3(\cos x)^2$

$= 3\cos^2 x$

and $fog(x) = f(g(x)) = f(3x^2) = \cos 3x^2$

Sol 5:- (i) Given $f(x) = |x|$ and $g(x) = |5x-2|$

$(gof)(x) = g(f(x)) = g(|x|) = |5|x|-2|$

$(fog)(x) = f(g(x)) = f(|5x-2|) = ||5x-2|| = |5x-2|$

(ii) Given $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$

$(gof)(x) = g(f(x)) = g(8x^3) = (8x^3)^{\frac{1}{3}} = 2x$

$(fog)(x) = f(g(x)) = f(x^{1/3}) = 8(x^{1/3})^3 = 8x$

Sol.6:→ Given $f(x) = \frac{4x+3}{6x-4}$

$(f\circ f)(x) = f(f(x)) = f\left(\frac{4x+3}{6x-4}\right)$

$= \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} = \frac{16x+12+18x-12}{24x+18-24x+16}$

$= \frac{34}{34}x = x$

$\therefore (f\circ f)(x) = x = I(x)$

$f^{-1} = f$

Sol.7:→ $f(x) = \frac{x}{x+2}$

$D_f = R - \{-2\}$

To find Range of f: Let $y = f(x)$

$\Rightarrow y = \frac{x}{x+2}$

$\Rightarrow xy + 2y = x \Rightarrow x = \frac{2y}{1-y}$

For Real x, $y \neq 1$

$R_f = R - \{1\}$

Injectivity :- Let $x_1, x_2 \in D_f$

$\therefore f(x_1) = f(x_2) \Rightarrow \frac{x_1}{x_1+2} = \frac{x_2}{x_2+2}$

$\Rightarrow x_1x_2 + 2x_1 = x_1x_2 + 2x_2$

$\Rightarrow x_1 = x_2$

$\therefore f(x_1) = f(x_2)$ for $x_1 = x_2$

To find the inverse of f: Let $y = f(x)$

$\Rightarrow y = \frac{x}{x+2} \Rightarrow x = \frac{2y}{1-y}$

$\Rightarrow f^{-1}(y) = \frac{2y}{1-y}$

i.e $f^{-1}(x) = \frac{2x}{1-x}$ $\forall x \in D_{f^{-1}} = R - \{1\}$

Sol.8:- Let $x_1, x_2 \in R$

$\therefore f(x_1) = f(x_2) \Rightarrow 4x_1+3 = 4x_2+3$

$\Rightarrow 4x_1 = 4x_2$

$\therefore x_1 = x_2$

$\therefore f$ is one-one

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Also let $y \in R_f$, then $y = f(x)$

$y = 4x+3$

$x = \frac{y-3}{4} \in R$

$\therefore$ the function is on to

Thus the function is one-one and onto

To find the Inverse of f:

As $f\left(\frac{y-3}{4}\right) = y$

$\frac{y-3}{4} = f^{-1}y$

$\Rightarrow f^{-1}(x) = \frac{x-3}{4}$

Sol. 9:- Let x_1, x_2 be any two elements of R^+ such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^2+4 = x_2^2+4 \Rightarrow x_1^2 = x_2^2$

$\Rightarrow |x_1| = |x_2|$

But both x_1 and x_2 are positive real no.

$\therefore x_1 = x_2$

So $f: R^+ \to [4, \infty)$ is one-one.

Also: Let $y \in [4, \infty)$, then $y = f(x)$

$\Rightarrow y = x^2+4 \Rightarrow x^2 = y-4$

$\Rightarrow x = \sqrt{y-4} \in R^+$

Hence f is both one-one and on to

$\Rightarrow f^{-1}$ exist.

To find inverse of f: As $f(\sqrt{y-4}) = y$

$\Rightarrow \sqrt{y-4} = f^{-1}(y) \Rightarrow f^{-1}(x) = \sqrt{x-4}$

Sol. 10:- Let x_1, x_2 be any two elements of R^+ s.t $f(x_1) = f(x_2)$

$\Rightarrow 9x_1^2+6x_1-5 = 9x_2^2+6x_2-5$

$\Rightarrow 9x_1^2+6x_1 = 9x_2^2+6x_2$

$\Rightarrow 9(x_1^2-x_2^2)+6(x_1-x_2) = 0$

$\Rightarrow 3(x_1-x_2)+[3(x_1+x_2)+2] = 0$

$\Rightarrow x_1 - x_2 = 0$ $\quad [\because 3(x_1+x_2)+2 \neq 0$ as $x_1, x_2 \geq 0$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one-one

Surjectivity :- Let y be any element of $[-5, \infty)$

then $y = f(x) \Rightarrow y = 9x^2 + 6x - 5$

$\Rightarrow 9x^2 + 6x - 5 - y = 0$

$$\Rightarrow x = \frac{-6 \pm \sqrt{36 + 36(5+y)}}{18}$$

$$= \frac{-6 \pm \sqrt{36(y+6)}}{18} = \frac{-1 \pm \sqrt{y+6}}{3}$$

Since $x \geq 0$, Therefore $x = \frac{-1 - \sqrt{y+6}}{3}$ is not possible

$$\therefore x = \frac{-1 + \sqrt{y+6}}{3} \geq 0$$

$\Rightarrow \sqrt{y+6} \geq 1 \Rightarrow y \geq -5$

$\therefore R_f = [-5, \infty) =$ co-domain

$\Rightarrow f$ is onto

$\therefore f$ is one-one & onto both hence f^{-1} exist.

To find inverse of f :

$$y = f(x)$$

$$\Rightarrow x = \frac{-1 + \sqrt{y+6}}{3} \Rightarrow y = f\left(\frac{-1 + \sqrt{y+6}}{3}\right)$$

$$\Rightarrow f^{-1}(y) = \frac{-1 + \sqrt{y+6}}{3}$$

$$\Rightarrow f^{-1}(x) = \frac{-1 + \sqrt{x+6}}{3}$$

Sol. 11 :- Here $f(x) = x^2 - 3x + 2$

$f(f(x)) = (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$

$= x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2 - 3x^2 + 9x - 6 + 2$

$= \cancel{x^4 + 10x}$

$= x^4 - 6x^3 + 10x^2 - 3x$

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Sol. 12:- $R = \{(a,b) : a \le b^2, a, b \in R\}$

Reflexive : As $\frac{1}{2} > \frac{1}{4}$, for $\frac{1}{2}, \frac{1}{4} \in R$

i.e $\frac{1}{2} \nleq (\frac{1}{2})^2$

$\therefore (a,a) \notin R \ \forall a \in R$

Hence R is not reflexive

Symmetric :- As $2, 5 \in R$ and $2 < 25$ i.e $2 < 5^2$

$\therefore (2,5) \notin R$

But $5 \nleq 4$ i.e $5 \nleq 2^2$

$\therefore (5,2) \in R$ $\therefore (2,5) \in R$ but $(5,2) \notin R$

Hence R is not symmtric

Transitive :- As $3, -2$ and $-1 \in R$

and $3 < (-2)^2$ and $-2 < (-1)^2$

$\therefore (3,-2) \in R$ and $(-2,-1) \in R$

But $3 \nleq (-1)^2 \Rightarrow (3,-1) \notin R$

$\therefore (3,-2) \in R$ and $(-2,-1) \in R$

$\Rightarrow (3,-1) \notin R$ Hence R is not transitive

$\therefore$ R is neither reflexive nor symmetric nor transitive.

Sol. 13:- Here $a * b = ab^2$, $a, b \in Q$

$\therefore b * a = ba^2$, $a, b \in Q$

$\Rightarrow a * b \neq b * a$

which shows that operation * is not commutative.

Sol. 14 :- $a * b = \frac{a+b}{2}$, where $a, b \in Q$.

Let $a, b, c \in Q$, By definition $a * b = \frac{a+b}{2}$, $(a, b \in Q)$

Now $a * b = \frac{a+b}{2} = \frac{b+a}{2}$ $(\because a, b \in Q)$

$= b * a$

$\Rightarrow a * b = b * a$

Hence operation * is commutative.

Again $(a*b)*c = \left(\frac{a+b}{2}\right) * c$ $(\because a, b \in Q)$

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$$\frac{\left(\frac{a+b}{2}\right)+c}{2} = \frac{a+b+2c}{4} \quad \left(\because \frac{a+b}{2}, c \in \mathbb{Q}\right)$$

and $a*(b*c) = a*\left(\frac{b+c}{2}\right) = \frac{a+\frac{b+c}{2}}{2}$

$$= \frac{2a+b+c}{4} \neq \frac{a+b+2c}{4}, \text{ in general}$$

$\Rightarrow \quad (a*b)*c \neq a*(b*c)$

Hence, operation $*$ is not associative.

Sol. 15 :- (a) Let $e \in \mathbb{Q}$ be the identity element of

$a*b = a^2+b^2$

$\Rightarrow \quad a*e = a^2+e^2 = a$

$\Rightarrow \quad e^2 = a-a^2$

Let $\quad a = 4 \in \mathbb{Q}$

$\Rightarrow \quad e^2 = 4-16 = -12$

$\Rightarrow \quad e = \pm\sqrt{-12} \notin \mathbb{Q}$

Hence, the binary operation $a*b = a^2+b^2, a,b \in \mathbb{Q}$ does not have an identity element.

(b) The binary operation is $a*b = a-b, a,b \in \mathbb{Q}$

Let $e \in \mathbb{Q}$ be its identity element

$\Rightarrow \quad a*e = e*a = a$

$\Rightarrow \quad a-e = a \quad -(i)$

or $\quad e-a = a \quad -(ii)$

Now $\quad a-e = a$

$\Rightarrow \quad e = 0$

Put in (ii), $0-a = a$, which is not true

Hence $a*b = a-b, a,b \in \mathbb{Q}$ does not have an identity element.

Note :-

Sol. 16 :- Let y be any arbitrary element of Y.
By the definition of Y, $y = 4x+3$ for some x in the domain N. This shows that $x = \frac{(y-3)}{4}$

Define $g: Y \rightarrow N$ by $g(y) = \frac{(y-3)}{4}$.

~~g(y)~~ Now $gof(x) = gf(x) = g(4x+3) = \frac{4x+3-3}{4} = x$

and $fog(y) = f(g(y)) = f\left(\frac{(y-3)}{4}\right) = \frac{4(y-3)}{4} + 3$

$= y-3+3 = y$. This shows that $gof = I_N$

and $fog = I_y$.

$\Rightarrow$ f is invertible and g is inverse of f.

Sol. 17:- Let y be an arbitrary element of range f.

Then $y = 4x^2+12x+15$, for some x in N.

$\Rightarrow y = (2x+3)^2+6$. This gives

$$x = \frac{((\sqrt{y-6})-3)}{2}, \text{ as } y \geq 6.$$

Let us defined $g: S \to N$ by $g(y) = \frac{((\sqrt{y-6})-3)}{2}$

Now $gof(x) = gf(x) = g(4x^2+12x+15) = g((2x+3)^2+6)$

$= \cancel{\left[\frac{((\sqrt{y-6})-3)}{2}\right]} \left[\frac{((2x+3)^2+6-6)-3}{2}\right]$

$= \frac{(2x+3-3)}{2} = x$

and $fog(y) = f\left[\frac{((\sqrt{y-6})-3))}{2}\right] = \left[2\frac{((\sqrt{y-6})-3)}{2} + 3\right]^2 + 6$

$= [((\sqrt{y-6})-3+3))]^2+6 = (\sqrt{y-6})^2+6$

$= y-6+6 = y$

Hence $gof = I_N$ and $fog = I_S$

$\Rightarrow$ f is invertible with $f^{-1} = g$

Chapter-2

Inverse Trigonometric functions

Sol. 1:- $L.H.S = \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

$$= \tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\times\frac{7}{24}}\right) \quad \left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \tan^{-1}\left(\frac{48+77}{11\times 24 - 2\times 7}\right)$$

$$= \tan^{-1}\left(\frac{125}{264-14}\right) = \tan^{-1}\left(\frac{125}{250}\right)$$

$$= \tan^{-1}\left(\tfrac{1}{2}\right) = R.H.S \text{ . Hence Proved}$$

Sol. 2 :- $L.H.S = 2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$

$$= \tan^{-1}\left(\frac{2\times\frac{1}{2}}{1-(\frac{1}{2})^2}\right) + \tan^{-1}\left(\frac{1}{7}\right) \quad \left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$= \tan^{-1}\left(\frac{1}{1-\frac{1}{4}}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right)$$

$$= \tan^{-1}\left(\frac{28+3}{21-4}\right) = \tan^{-1}\left(\frac{31}{17}\right) = R.H.S.$$

Sol. 3 :- (i) Let $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$$\therefore \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right) \quad \left[\because 1+\tan^2\theta = \sec^2\theta\right]$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\frac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2}\right)$$

$$= \tan^{-1}\left(\frac{\sin\theta/2}{\cos\theta/2}\right) = \tan^{-1}(\tan\theta/2)$$

$$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$$

which is required simplest form

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Sol. Q1 - (ii) $\tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$

Let $x = \operatorname{cosec}\theta \Rightarrow \theta = \operatorname{cosec}^{-1}x$

$$\therefore \tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{\cot^2\theta}}\right) = \tan^{-1}\left(\frac{1}{\cot\theta}\right)$$

$$= \tan^{-1}(\tan\theta) = \theta = \operatorname{cosec}^{-1}x$$

which is required simplest form

(iii) $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$

we write it as $\tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right)$

$$= \tan^{-1}\left(\frac{\sin x/2}{\cos x/2}\right) = \tan^{-1}\left(\tan\frac{x}{2}\right) = \frac{x}{2}$$

which is required simplest form

(iv) $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$

we write it as $\tan^{-1}\left(\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\right)$

$$= \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}-x\right)\right)$$

$\left[\because \tan\left(\frac{\pi}{4}-x\right) = \frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x} \text{ and } \tan\left(\frac{\pi}{4}\right)=1\right]$

$= \frac{\pi}{4} - x$. which is required simplest form.

(v) $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$

Let $x = a\sin\theta \Rightarrow \sin\theta = \frac{x}{a}$ and $\theta = \sin^{-1}\left(\frac{x}{a}\right)$

$$\therefore \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right) = \tan^{-1}\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}$$

$$= \tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right) = \tan^{-1}(\tan\theta)$$

$= \theta = \sin^{-1}\frac{x}{a}$. which is required simplest form.

Sol 4:- Let $\sin^{-1}\frac{3}{5} = \theta \Rightarrow \sin\theta = \frac{3}{5}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3}{4}$

$\Rightarrow \theta = \tan^{-1}\frac{3}{4}$

$\therefore$ L.H.S $= 2\sin^{-1}\frac{3}{5} = 2\theta = 2\tan^{-1}\frac{3}{4}$

$= \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-(\frac{3}{4})^2}\right)$ $\{\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= \tan^{-1}\left(\frac{\frac{3}{2}}{1-\frac{9}{16}}\right) = \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{7}{16}}\right) = \tan^{-1}\left(\frac{24}{7}\right) =$ R.H.S

Sol. 5:- Let $\sin^{-1}\frac{8}{17} = x$ and $\sin^{-1}\frac{3}{5} = y$

$\sin x = \frac{8}{17}$ and $\sin y = \frac{3}{5}$

$\therefore \cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\frac{64}{289}} = \sqrt{\frac{225}{289}} = \frac{15}{17}$

$\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\therefore \tan x = \frac{\sin x}{\cos x} = \frac{8}{15}$ and $\tan y = \frac{\sin y}{\cos y} = \frac{3}{4}$

we know

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$

$= \frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}} = \frac{32+45}{60-24} = \frac{77}{36}$

$\therefore x+y = \tan^{-1}\frac{77}{36}$

$\Rightarrow \sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

Sol. 6:- Let $\cos^{-1}\frac{4}{5} = x$ and $\cos^{-1}\frac{12}{13} = y$

$\Rightarrow \cos x = \frac{4}{5}$ and $\cos y = \frac{12}{13}$

$\sin x = \sqrt{1-\cos^2 x}$ and $\sin y = \sqrt{1-\cos^2 y}$

$\sin x = \sqrt{1-\frac{16}{25}}$ and $\sin y = \sqrt{1-\frac{144}{169}}$

$\sin x = \frac{3}{5}$ and $\sin y = \frac{5}{13}$

we know

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$= \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13}$$
$$= \frac{48-15}{65} = \frac{33}{65}$$
$$\therefore x+y = \cos^{-1}\frac{33}{65}$$
$$\Rightarrow \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$$

Sol 7:- Let $\cos^{-1}\frac{12}{13} = x$ and $\sin^{-1}\frac{3}{5} = y$

$$\Rightarrow \cos x = \frac{12}{13} \text{ and } \sin y = \frac{3}{5}$$
$$\sin x = \sqrt{1-\cos^2 x} = \sqrt{1 - \frac{144}{169}} = \frac{5}{13}$$
and $\cos y = \sqrt{1-\sin^2 y} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

we know that

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$= \frac{5}{13} \times \frac{4}{5} + \frac{12}{13} \times \frac{3}{5} = \frac{20+36}{65} = \frac{56}{65}$$
$$\therefore x+y = \sin^{-1}\frac{56}{65}$$
$$\Rightarrow \cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$$

Sol. 8:- Let $\sin^{-1}\frac{5}{13} = x$ and $\cos^{-1}\frac{3}{5} = y$

$$\Rightarrow \sin x = \frac{5}{13} \quad \text{and } \cos y = \frac{3}{5}$$
$$\therefore \cos x = \sqrt{1-\sin^2 x} = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$$
$$\cancel{\cos}\ \sin y = \sqrt{1-\cos^2 y} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

Also $\tan x = \frac{\sin x}{\cos x} = \frac{5}{12}$, $\tan y = \frac{\sin y}{\cos y} = \frac{4}{3}$

we know that

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\frac{5}{12} + \frac{4}{3}}{1 - \frac{5}{12} \times \frac{4}{3}}.$$

$$= \frac{\frac{5+16}{12}}{1-\frac{5}{9}} = \frac{\frac{21}{12}}{\frac{4}{9}}$$

$$= \frac{21}{12} \times \frac{9}{4} = \frac{7}{4} \times \frac{9}{4} = \frac{63}{16}$$

$$x+y = \tan^{-1}\frac{63}{16}$$

$$\Rightarrow \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5} = \tan^{-1}\frac{63}{16}.$$

Sol 9. Let $\tan\theta = x$

$$\Rightarrow \theta = \tan^{-1}x$$

Now $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2x}{1+x^2}$

$$\Rightarrow 2\theta = \sin^{-1}\left[\frac{2x}{1+x^2}\right]$$

$$\Rightarrow 2\tan^{-1}x = \sin^{-1}\left[\frac{2x}{1+x^2}\right]$$

Sol. 10:- Let $\tan^{-1}\sqrt{x} = \theta \Rightarrow \sqrt{x} = \tan\theta$

$$\Rightarrow x = \tan^2\theta$$

$$\therefore L.H.S = \tan^{-1}\sqrt{x} = \theta \quad -(1)$$

$$R.H.S = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$= \frac{1}{2}\cos^{-1}(\cos 2\theta) = \frac{1}{2}(2\theta) = \theta \quad -(2)$$

$\therefore$ from (1) & (2)

we get $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

Sol. 11:- Let $\sin^{-1}\left(\frac{1}{2}\right) = \theta$

$$\Rightarrow \sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

$$\therefore \tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] = \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right]$$

$$= \tan^{-1}\left[2\times\frac{1}{2}\right] = \tan^{-1}(1)$$

$$= \tan^{-1}\left[\tan\frac{\pi}{4}\right] = \frac{\pi}{4}$$

Thus $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] = \frac{\pi}{4}$

Sol. 12 :- L.H.S $= \tan^{-1}\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

put $x = \cos 2\theta$

$$= \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)$$

$$= \frac{\pi}{4}-\theta$$

$$= \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x = \text{R.H.S}$$

Sol. 13 :- L.H.S $= \cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

$$= \cot^{-1}\left[\frac{\sqrt{1+\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{1-\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{1+\cos\left(\frac{\pi}{2}-x\right)}-\sqrt{1-\cos\left(\frac{\pi}{2}-x\right)}}\right]$$

$$= \cot^{-1}\left[\frac{\sqrt{2}\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)+\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)}{\sqrt{2}\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)-\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$$

$$= \cot^{-1}\left(\frac{1+\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{1-\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$$

$$= \cot^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\pi}{4}-\frac{x}{2}\right)\right)$$

$$= \cot^{-1}\left(\tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right)$$

$$= \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$= \frac{x}{2}$$

Sol. 14 :→

$$L.H.S = \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8}$$

$$= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}}\right) + \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\times\frac{1}{8}}\right)$$

$$= \tan^{-1}\left(\frac{7+5}{35-1}\right) + \tan^{-1}\left(\frac{8+3}{24-1}\right)$$

$$= \tan^{-1}\left(\frac{12}{34}\right) + \tan^{-1}\left(\frac{11}{23}\right)$$

$$= \tan^{-1}\left(\frac{6}{17}\right) + \tan^{-1}\left(\frac{11}{23}\right)$$

$$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\times\frac{11}{23}}\right) = \tan^{-1}\left(\frac{138+187}{391-66}\right)$$

$$= \tan^{-1}\left(\frac{325}{325}\right) = \tan^{-1} 1 = \frac{\pi}{4} = R.H.S$$

Sol. 15 :-

$$\sin^{-1}\frac{12}{13} = \tan^{-1}\left[\frac{\frac{12}{13}}{\sqrt{1-\frac{144}{169}}}\right] \qquad \left[\because \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{12}{13}}{\frac{5}{13}}\right] = \tan^{-1}\frac{12}{5}$$

$$\cos^{-1}\frac{4}{5} = \tan^{-1}\left[\frac{\sqrt{1-\frac{16}{25}}}{\frac{4}{5}}\right] \qquad \left[\because \cos^{-1}x = \tan^{-1}\frac{\sqrt{1-x^2}}{x}\right]$$

$$= \tan^{-1}\left[\frac{\frac{3}{5}}{\frac{4}{5}}\right] = \tan^{-1}\frac{3}{4}$$

$$\therefore \sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \tan^{-1}\frac{12}{5} + \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{63}{16}$$

$$= \pi + \tan^{-1}\left[\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}\times\frac{3}{4}}\right] + \tan^{-1}\frac{63}{16}$$

$$= \pi + \tan^{-1}\left(-\frac{63}{16}\right) + \tan^{-1}\frac{63}{16}$$

$$= \pi - \tan^{-1}\frac{63}{16} - \tan^{-1}\frac{63}{16}$$

$$= \pi$$

Sol. 16. L.H.S $= \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right)$

$$= \tan^{-1}\left[\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}\times\frac{1}{5}}\right] + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left[\frac{\frac{7}{10}}{\frac{9}{10}}\right] + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{7}{9}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left[\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{9}\times\frac{1}{8}}\right] = \tan^{-1}\left[\frac{\frac{56+9}{72}}{\frac{72-7}{72}}\right]$$

$$= \tan^{-1}\left[\frac{65}{65}\right] = \tan^{-1}(1) = \frac{\pi}{4}$$

Sol. 17:- Given $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$

Let $\tan^{-1}x = \theta \Rightarrow x = \tan\theta$

$\therefore \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right) = \frac{1}{2}\theta$

$\Rightarrow \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right) = \frac{1}{2}\theta$

$\frac{\pi}{4} - \theta = \frac{1}{2}\theta$

$\Rightarrow \frac{3}{2}\theta = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{6}$

$\therefore x = \tan\theta = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

Sol. 18:- Let $\sin^{-1}\left(\frac{3}{5}\right) = x$ and $\sin^{-1}\left(\frac{8}{17}\right) = y$

$\Rightarrow \sin x = \frac{3}{5}$ and $\sin y = \frac{8}{17}$

$\Rightarrow \cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

$\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-\frac{64}{289}} = \frac{15}{17}$

Now $\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\Rightarrow \cos(x-y) = \frac{4}{5}\times\frac{15}{17} + \frac{3}{5}\times\frac{8}{17} = \frac{60}{85} + \frac{24}{85} = \frac{84}{85}$

$\Rightarrow x - y = \cos^{-1}\left(\frac{84}{85}\right)$

$\Rightarrow \sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$
