Interference of Light

Interference of light ↦ The phenomenon of non ~~distor~~ uniform distribution of energy due to super position of two light waves from two coherent sources is called interference of light.

When two ~~wa~~ light waves of same frequ... same wavelength, same velocity ~~a~~ and either hav... zero or constant phase difference while travellin... in same direction superimpose on each other we get phenomenon of interference.

In interference pattern alternative bright and dark fringes (bands ~~spots~~) are formed.

Constructive interference ↦ Due to superposition of light waves, at some points ~~bright~~ intensity of resultant light is maximum. Such interference is called constructive interference.

Destructive Interference ↦ Due to superposition of light waves, at some points intensity of resultant light is minimum. Such interference is called destructive interference.

Coherent Sources ↦ Two source are said to be coherent if they emit light waves of same frequency, same wavelength (and hence same velocity) and either have zero or constant phase differen... are called coherent sources.

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Principle of superposition of light waves ↦

It states that when two or more light waves superimpose on each other, then displacement vector of the resultant light wave is equal to the vector sum of displacements of different superimposing waves at that instant.

ie if $\vec{y}_1, \vec{y}_2, \vec{y}_3$ --- are the displacement vectors of the different superimposing waves at any instant, then displacement vector of the resultant wave at the same instant will be,

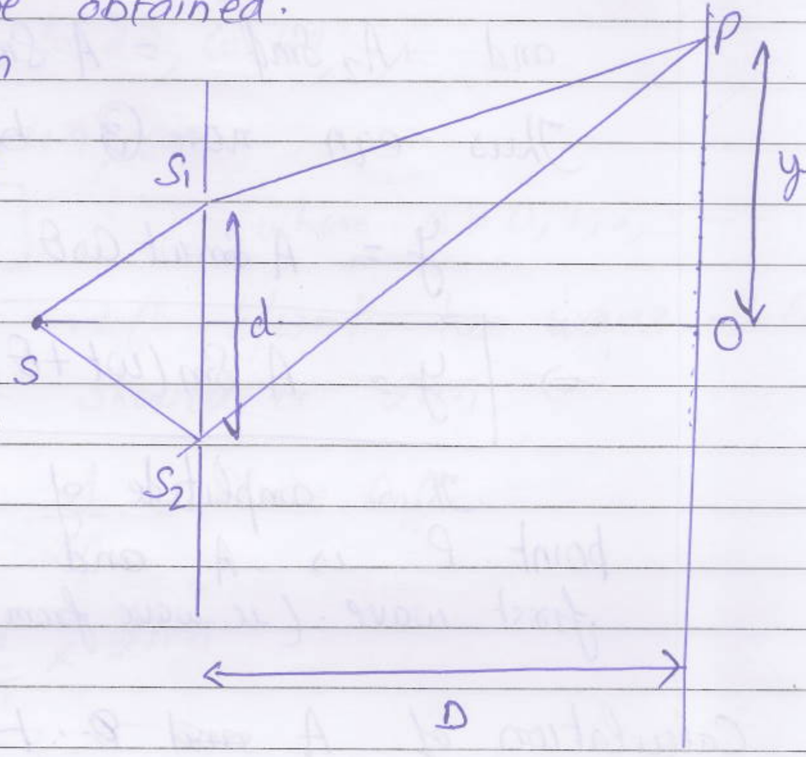
$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \text{---}$$

Young's Double Slit Experiment and condition for constructive and destructive interference ↦

In Young's double slit experiment a monochromatic source of light S is placed in front of two equidistant narrow slits S_1 and S_2. From Huygen's principle S_1 and S_2 also act as source of secondary disturbance. Since light at S_1 and S_2 are coming from the same source thus light at S_1 and S_2 is of same wavelength, same frequency and will have zero phase difference. ie S_1 and S_2 will act as coherent sources.

Now light will spread from S_1 and S_2 and will interfere with each other. Depending upon the path difference and hence phase difference between the two waves while reaching at a point on the screen either constructive or destructive interference will be obtained.

At the centre of screen the intensity of light is maximum and is called central maxima.



Conditions for C.I or D.I :-

Consider two waves reaches from two coherent sources S_1 and S_2 reach at any point P. Due to the path diff. b/w two waves while reaching at point, they must have phase diff.

Let ϕ is the phase difference b/w two waves while reaching at point P.

Thus displacement of waves at point P after time can be given as

$$y_1 = A_1 \sin \omega t \quad \text{---} (1)$$

and $$y_2 = A_2 \sin(\omega t + \phi) \quad \text{---} (2)$$

Thus from superposition principle, displacement of the resultant wave at time t will be:

$$y = y_1 + y_2$$

$$= A_1 \sin \omega t + A_2 \sin(\omega t + \phi) \qquad (\text{using } (1) \text{ and } (2))$$

$$= A_1 \sin \omega t + A_2 [\sin \omega t \cos \phi + \cos \omega t \sin \phi]$$

$$y = (A_1 + A_2 \cos \phi) \sin \omega t + A_2 \sin \phi \cos \omega t \quad \text{---} (3)$$

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Let $A_1 + A_2 \cos\phi = A\cos\theta$ —— ④

and $A_2 \sin\phi = A\sin\theta$ —— ⑤

Thus eqn no. ③ becomes

$$y = A\sin\omega t\cos\theta + A\cos\omega t\sin\theta$$

$$\Rightarrow \boxed{y = A\sin(\omega t + \theta)}$$

Thus amplitude of the resultant wave at point P is A and has θ phase diff. wrt first wave. (ie wave from S_1)

Calculation of A ~~and~~ ~~θ~~. ↦

Squaring and adding eqn no ④ and ⑤ we get

$$(A_1 + A_2\cos\phi)^2 + (A_2\sin\phi)^2 = A^2\cos^2\theta + A^2\sin^2\theta$$

$$\Rightarrow A^2(\cos^2\theta + \sin^2\theta) = A_1^2 + A_2^2\cos^2\phi + 2A_1A_2\cos\phi + A_2^2\sin^2\phi$$

$$\Rightarrow A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi \quad \text{—— ⑥}$$

$$\Rightarrow A = \sqrt{A_1^2 + A_2^2 + 2A_1A\cos\phi}.$$

For constructive Interference ↦ For C.I., intensity of the resultant wave should be max.

Since $I \propto A^2$

Thus I will be max.

when A^2 is max

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from ie $\cos\phi = 1$ (from eqn no ⑥)

$\Rightarrow \cos\phi = \cos 0, \cos 2\pi, \cos 4\pi, ---$

$\Rightarrow \phi = 0, 2\pi, 4\pi, 6\pi ---$

$\boxed{\phi = 2n\pi}$ where $n = 0, 1, 2, 3 ---$

ie for C.I., phase diff. b/w the two waves while reaching at point P should be $2n\pi$.

Also path diff $= \frac{\lambda}{2\pi} \times$ Phase Diff.

$\therefore \Delta x = \frac{\lambda}{2\pi} \times 2n\pi$

$\Rightarrow \boxed{\Delta x = n\lambda}$ for; $n = 0, 1, 2, 3 ---$

ie for C.I, path diff. b/w the two waves while reaching at point P should be $n\lambda$.

For Destructive Interference ↦ for D.I, intensity of resultant wave should be minimum

Since $I \propto A^2$

$\therefore$ I will be minimum

when A^2 is minimum

ie $\cos\phi = -1$ (from eqn no ⑥)

$\Rightarrow \cos\phi = \cos\pi, \cos 3\pi, \cos 5\pi, ---$

$\Rightarrow \phi = \pi, 3\pi, 5\pi, ---$

$\Rightarrow \boxed{\phi = (2n+1)\pi}$ for $n = 0, 1, 2, 3 ---$

ie for D.I, phase diff b/w the two interfering waves while reaching at point P should be $(2n+1)\pi$.

and path differnce,

$$\Delta x = \frac{\lambda}{2\pi}(2n+1)\pi$$

$$\Rightarrow \boxed{\Delta x = (2n+1)\frac{\lambda}{2}}$$

ie for destructive interference, path diff b/w the two interfering wave while reach at any point P should be $(2n+1)\frac{\lambda}{2}$.

For C. I. $\cos\phi = 1$

$\therefore A^2 = A_1^2 + A_2^2 + 2A_1A_2$ (from ⑥)

$$\Rightarrow \boxed{A = A_1 + A_2}$$

if $A_1 = A_2 = A'$ says

then $A = 2A'$

and $I_{max} = K(2A')^2$

for D. I. $\cos\phi = -1$

$\therefore A^2 = A_1^2 + A_2^2 - 2A_1A_2$ (from 6)

$$\Rightarrow \boxed{A = A_1 - A_2}$$

if $A_1 = A_2 = A'$ (say)

then $A = 0$

$\therefore I_{min} = 0.$

ie Dark fringe will be compeletely dark.

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Ratio of intensities of light at maxima and minima ⟼

we know that

$$I \propto A^2$$

for C.I $\quad A = A_1 + A_2$

$$\therefore I_{max} \propto (A_1 + A_2)^2$$

for D.I $\quad A = A_1 - A_2$

$$\therefore I_{min} \propto (A_1 - A_2)^2$$

$$\therefore \frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

$$= \frac{A_1^2 (1 + A_2/A_1)^2}{A_1^2 (1 - A_2/A_1)^2}$$

$$\Rightarrow \boxed{\frac{I_{max}}{I_{min}} = \frac{(1+r)^2}{(1-r)^2}}$$

where $r = A_2/A_1$ = Amplitude ratio,

Intensity of light and width of the slit ⟼

Intensity of light from a slit is directly proportional to width of the slit.

ie $\quad I_1 \propto \omega_1 \quad$ (for slit S_1)

and $\quad I_2 \propto \omega_2 \quad$ (" " S_2)

$$\Rightarrow \boxed{\frac{I_1}{I_2} = \frac{\omega_1}{\omega_2}}$$

also $\quad I_1 \propto A_1^2$

$\quad I_2 \propto A_2^2$

$$\Rightarrow \boxed{\frac{I_1}{I_2} = \frac{A_1^2}{A_2}}$$

Thus $\boxed{\frac{\omega_1}{\omega_2} = \frac{A_1^2}{A_2}}$

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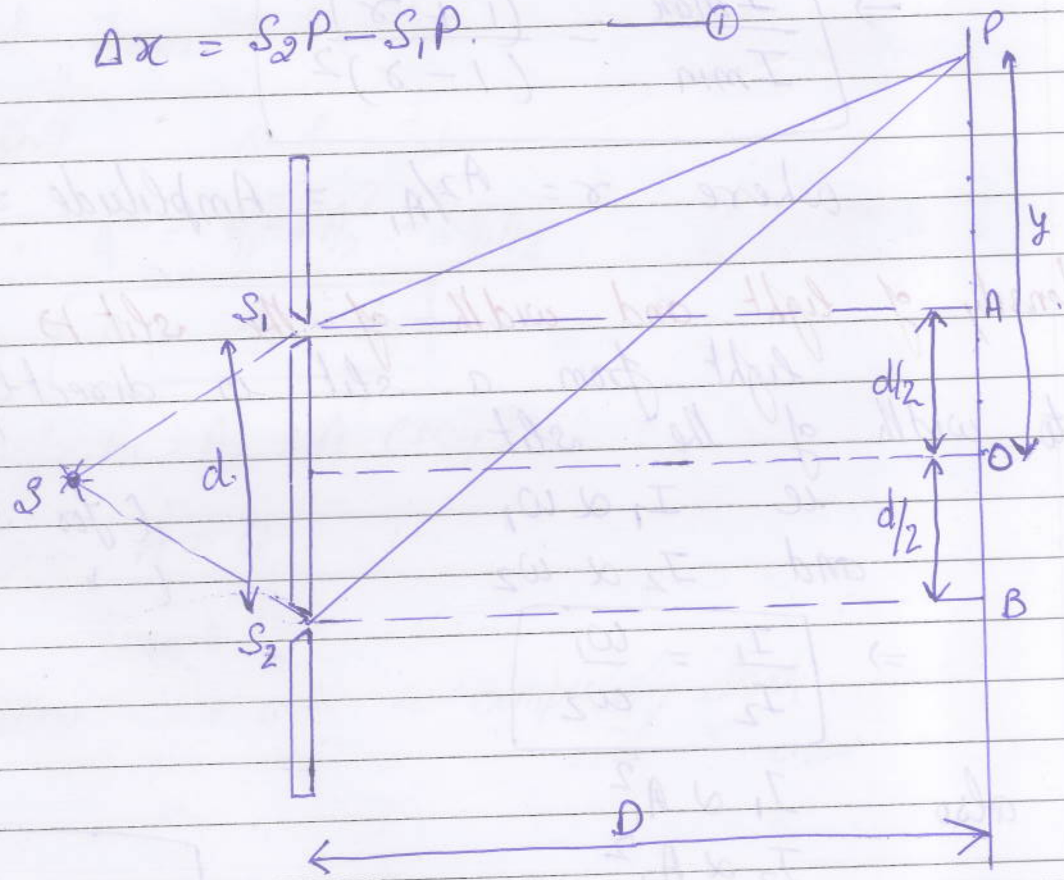
Positions of maxima and Minimas and Expression for fringe width ↦

Consider two coherent sources S_1 and S_2 are placed at a distance d from each other. A screen is placed at a ⊥ distance D from the sources. The waves from the two coherent sources interfere with each other and depending upon the path difference ~~b/w the two waves~~ and ~~phase~~ hence phase diff. b/w the two wave while reaching at ~~po~~ a point on the screen either bright or dark fringe is formed.

Consider any arbitrary point P on the screen at a distance y from the centre of screen. Thus the path difference b/w the two waves while reaching at point P will be

$$\Delta x = S_2P - S_1P \quad \text{———} (1)$$



where $S_2P = (S_2B^2 + BP^2)^{1/2}$

$$= \left[D^2 + \left(y + \frac{d}{2} \right)^2 \right]^{1/2}$$

$$= D\left[1 + \frac{(y + d/2)^2}{D^2} \right]^{1/2}$$

$$S_2P = D\left[1 + \frac{1}{2}\frac{(y+d/2)^2}{D^2} \right] \quad \text{(using Binomial Theorem)}$$

Similarly $S_1P = D\left[1 + \left(\frac{y - d/2}{D^2} \right)^2 \right]$

$\therefore$ Path Diff $\Delta x = D\left[1 + \frac{1}{2}\frac{(y+d/2)^2}{D^2} - 1 - \frac{(y-d/2)^2}{D^2} \right]$

$$= \frac{1}{2D}\left(y^2 + \frac{d^2}{4} + yd - y^2 - \frac{d^2}{4} + yd \right)$$

$$\Rightarrow \Delta x = \frac{1}{2D} \times 2yd$$

$$\Rightarrow \Delta x = \frac{yd}{D} \quad \text{——— (2)}$$

For bright fringes $\mapsto$ (for constructive interference)

~~Path Diff = $\frac{yd}{D}$~~

$$\text{Path diff} = n\lambda \qquad \text{for } n = 0, 1, 2, 3 \dots$$

$$\Rightarrow \frac{yd}{D} = n\lambda \qquad \{\text{using (2)}\}$$

$$\Rightarrow y = \frac{nD\lambda}{d} \qquad \text{for } n = 0, 1, 2, 3 \dots$$

Thus by substituting the values of n, we will have the different positions of bright fringes.

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for $n = 0$, $y_0 = 0$ — position of central maxima.

for $n = 1$, $y_1 = \frac{1 D\lambda}{d}$ " " first Bright fringe.

for $n = 2$, $y_2 = \frac{2 D\lambda}{d}$ " " 2^{nd} " "

for $n = n$, $y_n = \frac{n D\lambda}{d}$ " " n^{th} " "

For destructive interference or dark fringes ↦

$$\text{Path diff} = (2n+1)\frac{\lambda}{2} \quad \text{for } n = 0, 1, 2, 3 -$$

$$\Rightarrow \frac{yd}{D} = (2n+1)\frac{\lambda}{2} \quad (\text{using } 2)$$

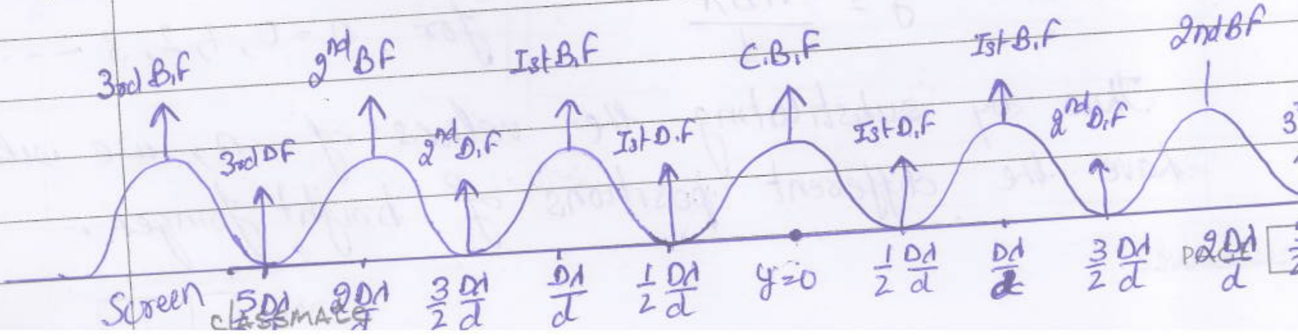
$$\Rightarrow y = (2n+1)\frac{D\lambda}{2d} \quad \text{for } n = 0, 1, 2, 3 -$$

for $n = 0$, $y'_0 = \frac{1}{2}\frac{D\lambda}{d}$ Position of Ist Dark fringe

for $n = 1$, $y'_1 = \frac{3}{2}\frac{D\lambda}{d}$ " " 2^{nd} " "

for $n = 2$, $y'_2 = \frac{5}{2}\frac{D\lambda}{d}$ " " 3^{rd} " "

for $n = n$, $y'_n = (2n+1)\frac{D\lambda}{2d}$ " " $(n+1)^{th}$ " "



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Fringe width of dark fringe ↦ The distance between two consecuti[ve] bright fringes is equal to fringe width of dark fring[e]

ie $\beta = y_n - y_{n-1}$

$= \frac{nD\lambda}{d} - (n-1)\frac{D\lambda}{d}$

$= \frac{D\lambda}{d}(n-n+1)$

$$\Rightarrow \boxed{\beta = \frac{D\lambda}{d}} \text{ ——— } (3)$$

Fringe width of ~~for~~ bright fringes ↦ The distance between t[wo] consecutive dark fringes is equal to fringe width of ~~dark~~ bright fringe.

ie $\beta' = y'_n - y'_{n-1}$

$= (2n+1)\frac{D\lambda}{2d} - [2(n-1)+1]\frac{D\lambda}{2d}$

$= \frac{D\lambda}{2d}(2n+1-2n+2-1)$

$= \frac{D\lambda}{2d} \times 2$

$$\Rightarrow \boxed{\beta' = \frac{D\lambda}{d}} \text{ ——— } (4)$$

from eqn no. (3) and (4) it is clear that width of dark fringes is equal to width of bright fringes.

Note ↦ Since $\beta \propto \lambda$, ∴ for blue colour fringe width w[ill] be small and for red colour la[rge].

Also for β to be large, D should be large and d should be small.

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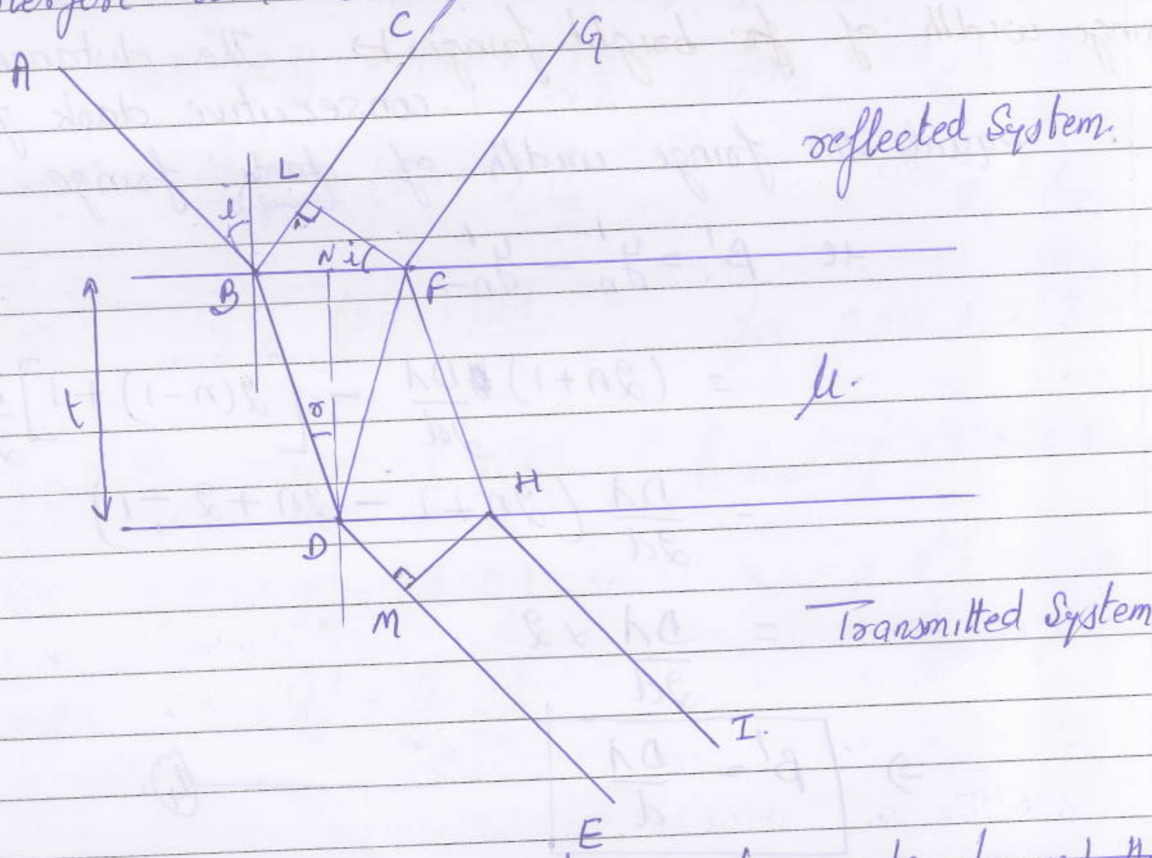
Interference due to thin films ⇒ Consider a thin transparent film of thickness t and refractive index μ. A monochromatic ray of light AB is incident on it at angle of incidence i. At point B a part of ray get refracted and a part get reflected. Similarly at pts D, F and H, a part get refracted and a part get reflected. Thus rays BC and FG interfere with each other in reflected system and rays DE and HI interfere with each other in transmitted system.



~~From the simple geomatery, it can be proved that~~

The geomaterical path diff. b/w the two rays i.e BC and FG interfering in the reflected system is.,

$$x' = \mu(BD + DF) - BL$$

By using simple geomatery, it can be proved that

$$x' = 2\mu t \cos r$$

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Since ray BC has been reflected by the denser med., therefore it has traversed an additional path of

Thus net path Diff is

$$x = 2\mu t \cos r - \lambda/2$$

for C.I. (Bright fringe)

$$x = n\lambda$$

$$\Rightarrow 2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow 2\mu t \cos r = n\lambda + \frac{\lambda}{2}$$

$$\boxed{2\mu t \cos r = (2n+1)\frac{\lambda}{2}} \quad \text{— (1)}$$

and for D.I (Dark fringe)

$$x = (2n+1)\lambda/2$$

$$\Rightarrow 2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow 2\mu t \cos r = (2n+1)\frac{\lambda}{2} + \frac{\lambda}{2}$$

$$\Rightarrow 2\mu t \cos r = (2n+2)\frac{\lambda}{2}$$

$$= (n+1)\lambda$$

(2) for n=

or $\boxed{2\mu t \cos r = n\lambda}$ —

for n=

The geomaterical path difference b/w the two is DE and HI interfering in the transmitted system is

$$x' = \mu(DF + FH) - DM$$

Illy $\quad x' = 2\mu t \cos r$

Since neither DE nor HI is reflected from the denser med., therefore there will be no additional path difference.

Thus net path diff.

$$x = x' = 2\mu t \cos r$$

for C.I or Bright Fringe

$$x = n\lambda$$

$$\Rightarrow \boxed{2\mu t \cos r = n\lambda} \quad \text{— (3)}$$

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and for D.I or dark fringe

$$x = (2n+1)\lambda/2$$

$$\Rightarrow \boxed{2\mu t \cos r = (2n+1)\frac{\lambda}{2}} \longrightarrow (4)$$

From eqn. no. (1), (2), (3) and (4) it is clear that the condition for C.I or bright fringe in the reflected system is same as that of condition for dark fringe or D.I in transmitted system and vice versa.

Thus the film which will appear bright in reflected system will appear dark in ~~transmit~~ the transmitted system and vice versa.

Colours in thin film ↦ We know that thin film will appear bright or dark depending on the factor that,

$$2\mu t \cos r = n\lambda \quad \text{ie } 1\lambda, 2\lambda, 3\lambda - - -$$

$$\text{or } 2\mu t \cos r = (2n+1)\frac{\lambda}{2} \quad \text{ie } \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda - - -$$

Since white light contains light of different wave length ranging from 3800 A° to 7600 A°. ~~That~~ Thus for certain wavelength $2\mu t \cos r$ may be equal to $n\lambda$ and for certain other wave lengths $2\mu t \cos r$ may be equal to $(2n+1)\frac{\lambda}{2}$.

Thus ~~the wave~~ for some wavelengths condition of bright fringes is fullfilled ~~and for~~ w/a other wavelength will satisfy the conditions of dark fringes. Thus we get certain colours in reflected system and other colours in transmitted system.

Interference and conservation of energy ↦

Consider two light sources, which emit light of amplitude a_1 and a_2 resp.

Thus intensity of light of source 1 is.

$$I_1 = a_1^2$$

llly $$I_2 = a_2^2$$

$\therefore$ Total Intensity $I = I_1 + I_2$

$$\Rightarrow I = a_1^2 + a_2^2 \quad \text{——— (1)}$$

If the light from two sources interfere with each other then maxima and minima are formed on the screen.

where $I_{max} = (a_1 + a_2)^2$

and $I_{min} = (a_1 - a_2)^2$

Thus net intensity of two sources

$$I = \frac{I_{max} + I_{min}}{2}$$

$$= \frac{(a_1+a_2)^2 + (a_1-a_2)^2}{2}$$

$$= \frac{a_1^2 + a_2^2 + 2a_1a_2 + a_1^2 + a_2^2 - 2a_1a_2}{2}$$

$$= \frac{2(a_1^2 + a_2^2)}{2}$$

$$\Rightarrow I = a_1^2 + a_2^2 \quad \text{——— (2)}$$

from ① and ② it is clear that there is no intensity loss or gain due to interference of light.

Hence law of conservation of energy is obeyed in the phenomenon of interference of light.

Conditions for sustained interference ↦

① Two source should be coherent. ie of same wavelength, same frequency and either have zero or constant phase difference.

2. Amplitude of the two waves should preferably be equal. Because for equal amplitude intensity of dark fringes will be zero. ie dark fringes will be completely dark.

3. Two light sources should be narrow. Because otherwise light from different portions of the same source will interfere differently and their interference pattern may overlap to produce general illumination.

4. For better interference pattern fringe width should be sufficiently large. ($\because \beta = \frac{D\lambda}{d}$)

Thus distance b/w the two source (d) should be very small.

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Condition for constructive and destructive interfer[ence]

In terms of path difference.

Path Diff. $\boxed{\Delta x = n\lambda}$

ie $\Delta x = 0, 1\lambda, 2\lambda, 3\lambda$ ---] for C.I.

$\boxed{\Delta x = (2n+1)\frac{\lambda}{2}}$

ie $\Delta x = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda$] for D.I.

In terms of phase diff.

we know that

Phase Diff. $\phi = \frac{2\pi}{\lambda} \cdot$ path diff.

$\Rightarrow \phi = \frac{2\pi}{\lambda}\Delta x$

Thus for Constructive Interference.

$$\phi = \frac{2\pi}{\lambda} \times n\lambda$$

$\Rightarrow \boxed{\phi = 2n\pi}$

ie $\phi = 0, 2\pi, 4\pi, 6\pi$ ---] for C.I

for Destructive interference

$$\phi = \frac{2\pi}{\lambda}(2n+1)\frac{\lambda}{2}$$

$\Rightarrow \boxed{\phi = (2n+1)\pi}$

ie $\phi = 1\pi, 3\pi, 5\pi,$ ---] for D.I.