Electrostatic potential and Capacitance.

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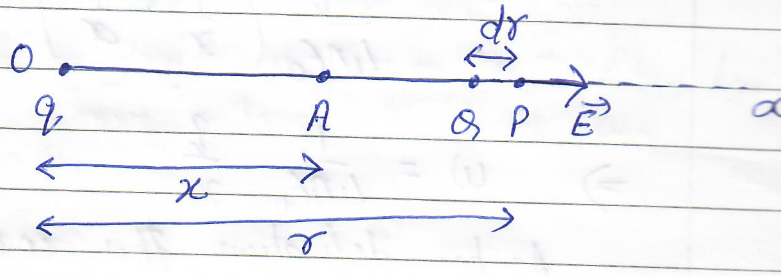
Electrostatic potential ↦ Electrostatic pot. at a point is dfa total amount of workdone in bringing the unit +ve charg from ∞ to that point in the electrostatic field of any o charge.

Expression ↦



Consider a pt. charge q at any pt. O. Suppose we want to find out the Electrostatic pot at any pt A in the electric field of q lying at a distance x from q.

Suppose a point P at distance r from q. Thus EF at pt. P due to charge q is.

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad \text{—— (1)}$$

Thus small amount of W.D. in bringing a unit +ve charge from P to Q through a small distance dr

$$dW = \vec{F}.\vec{dr}$$

$$= \vec{E}.\vec{dr} \qquad \left\{\because \vec{E} = \frac{\vec{F}}{q_0} \text{ and here } q_0 = 1\right.$$

$$= E\,dr\cos 180$$

$$= -E\,dr$$

$$= -\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}dr$$

Thus total WD in bringing the unit +ve charge from ∞ to A is.

$$W = \int_\infty^A dW = \int_\infty^x -\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}dr$$

$$= -\frac{q}{4\pi\epsilon_0}\int_\infty^x r^{-2}dr$$

$$\Rightarrow \quad W = \frac{-q}{4\pi\epsilon_0}\left|\frac{r^{-1}}{-1}\right|_{\infty}^{x}$$

$$= \frac{q}{4\pi\epsilon_0}\left|\frac{1}{r}\right|_{\infty}^{x}$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{x} - \frac{1}{\infty}\right)$$

$$\Rightarrow \quad W = \frac{1}{4\pi\epsilon_0}\frac{q}{x} \qquad \left\{\because \frac{1}{\infty} = 0\right.$$

As per defination this WD is equal to electrostatic pot. at pt. A.

ie $\boxed{V_A = \frac{1}{4\pi\epsilon_0}\frac{q}{x}}$

or $\text{Electric pot} = \frac{1}{4\pi\epsilon_0}\frac{\text{charge}}{\text{Distance}}$

Since V = Workdone.

$\therefore$ Electric pot is a scalar quantity.

Electrostatic pot. Difference ↦

Electrostatic pot. diff. b/w two points is d/o the total WD in moving a unit +ve charge from one point to another point in the electrostatic ~~pot. of~~ field of any other charge.

Expression ↦

Consider a point charge q at any point O. In order to find the Electrostatic pot. diff. b/w point A and B, consider a point P at distance r from the charge q as shown.

Thus EFI at pt. P is given as

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad \text{---}(1)$$

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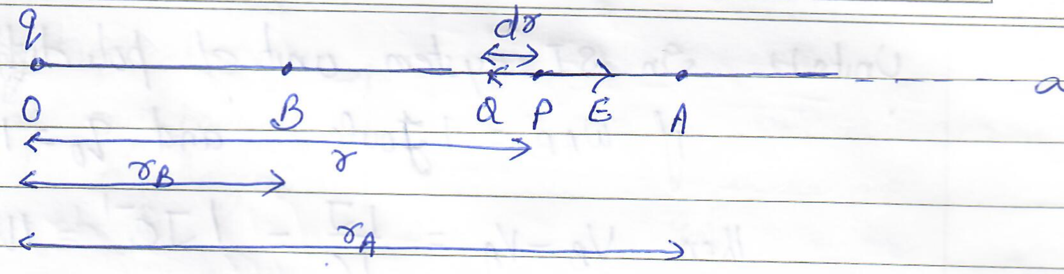

Thus small amount of WD in moving the unit +ve charge from pt P to Q is

$$dW = \vec{F} \cdot d\vec{r} = \vec{E} \cdot d\vec{r} \qquad \left\{ \because \vec{E} = \frac{\vec{F}}{q_0} \text{ \& here } q_0 = \right.$$

$$\Rightarrow dW = E\,dr \cos 180$$

$$= -E\,dr$$

$$dW = -\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} dr \qquad \{\text{using } ①\}$$

Thus total WD in moving the unit +ve charge from point A to B is

$$\frac{W_{AB}}{q_0} = \int_{r_A}^{r_B} -\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} dr$$

$$= -\frac{q}{4\pi\epsilon_0} \int_{r_A}^{r_B} r^{-2} dr$$

$$= -\frac{q}{4\pi\epsilon_0} \left| \frac{r^{-1}}{-1} \right|_{r_A}^{r_B}$$

$$= \frac{q}{4\pi\epsilon_0} \left| \frac{1}{r} \right|_{r_A}^{r_B}$$

$$= \frac{q}{4\pi\epsilon_0} \left| \frac{1}{r_B} - \frac{1}{r_A} \right|$$

$$\Rightarrow \frac{W_{AB}}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r_B} - \frac{1}{4\pi\epsilon_0} \frac{q}{r_A}$$

$$\Rightarrow \boxed{\frac{W_{AB}}{q_0} = V_B - V_A}$$

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Units ⟶ In SI system unit of pot. diff. is volt.

if $\omega_{AB} = 1$ Joule and $q_0 = 1C$

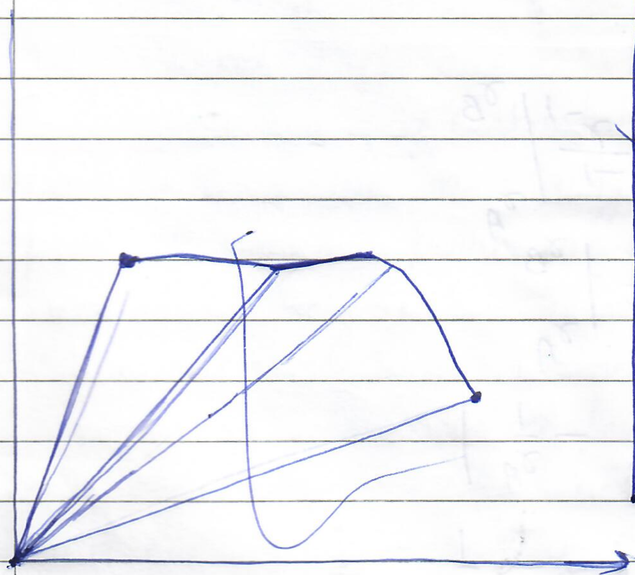
then $V_B - V_A = \frac{1J}{1C} = 1JC^{-1} = 1V.$

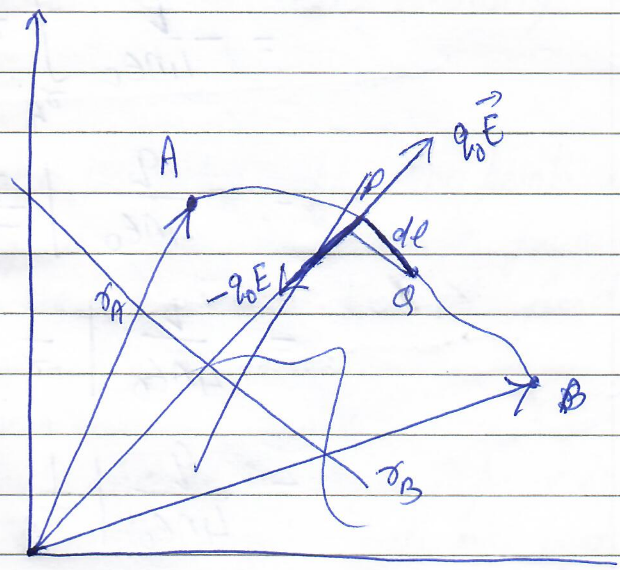
Thus pot. diff b/w two points is said to be 1V if 1J of work is done in moving 1C of charge from one point to another point.

Question ⟶ Show that WD in moving a unit +ve charge from one point to another point against ~~in~~ the electrostatic field on any other charge is independent of path followed and depends only upon positions of the two points.

or.

Show that line integral of EFI b/w two pts is independent of the shape of the line and depends only upon initial and final point of the line.

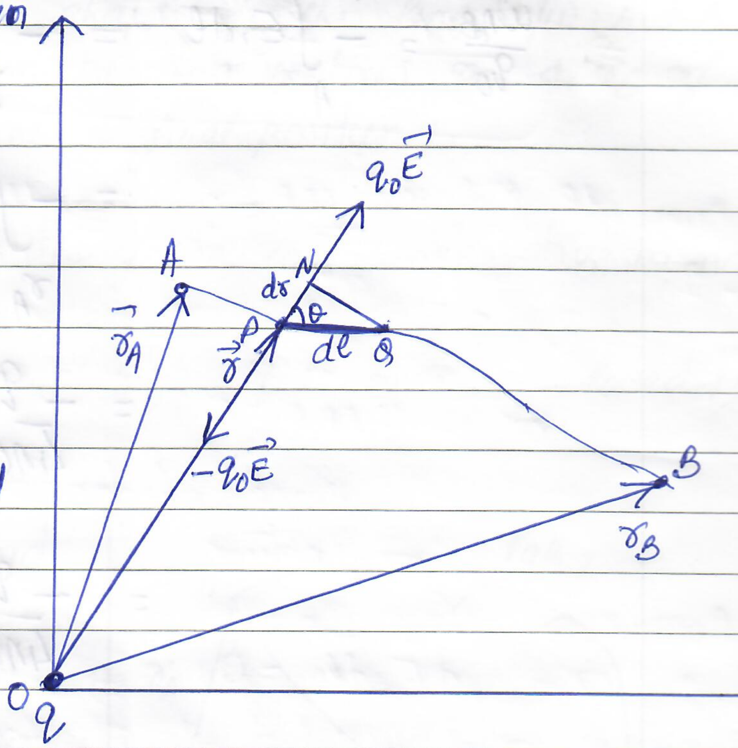




Consider a charge q at Origin O. The EFI at any point P having PV r due to charge q is

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad \text{—(1)}$$

$q_0\vec{E}$ is force exerted on any test charge q_0 at pt P. Thus $-q_0\vec{E}$ will be the required force to move the test charge q_0 against the Electrostatic field of charge q.



Thus small amount of WD in moving the test charge q_0 against the electrostatic field of q from pt P to Q will

$$dW = \vec{F}\cdot\vec{d\ell}$$
$$= -q_0\vec{E}\cdot\vec{d\ell}$$

Thus total WD in moving the test charge q_0 from point A to B against the electrostatic field of charge q

$$W_{AB} = \int_A^B -q_0\vec{E}\cdot\vec{d\ell}$$

or Total WD in moving a unit +ve charge from A to B against the electrostatic field is

$$\frac{W_{AB}}{q_0} = -\int_A^B \vec{E}\cdot\vec{d\ell} = -\int_A^B E\,d\ell\cos\theta \quad \text{—(2)}$$

In rt. angle Δ PQN

$$\frac{PN}{PQ} = \cos\theta \Rightarrow \frac{dr}{d\ell} = \cos\theta$$

$$\Rightarrow dr = d\ell\cos\theta$$

Hence eqn (2) becomes.

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$$\frac{W_{AB}}{q_0} = -\int_A^B \vec{E}\cdot\vec{dl} = -\int_A^B E\,dr$$

$$= -\int_{r_A}^{r_B} \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr$$ {using

$$= -\frac{q}{4\pi\epsilon_0}\int_{r_A}^{r_B} r^{-2}\,dr$$

$$= -\frac{q}{4\pi\epsilon_0}\left|\frac{r^{-1}}{-1}\right|_{r_A}^{r_B}$$

$$= \frac{q}{4\pi\epsilon_0}\left|\frac{1}{r}\right|_{r_A}^{r_B}$$

so $$\boxed{\frac{W_{AB}}{q_0} = -\int_A^B \vec{E}\cdot\vec{dl} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)}$$

for Ques 1
$\Rightarrow$ $$\frac{W_{AB}}{q_0} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

ie workdone in moving a unit +ve charge from one point to another point is independent of the pa[th] point A & B and depends only upon their positio[n] vectors (ie positions)

for Ques 2.

$$\Rightarrow -\int_A^B \vec{E}\cdot\vec{dl} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$\Rightarrow \int_A^B \vec{E}\cdot\vec{dl} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

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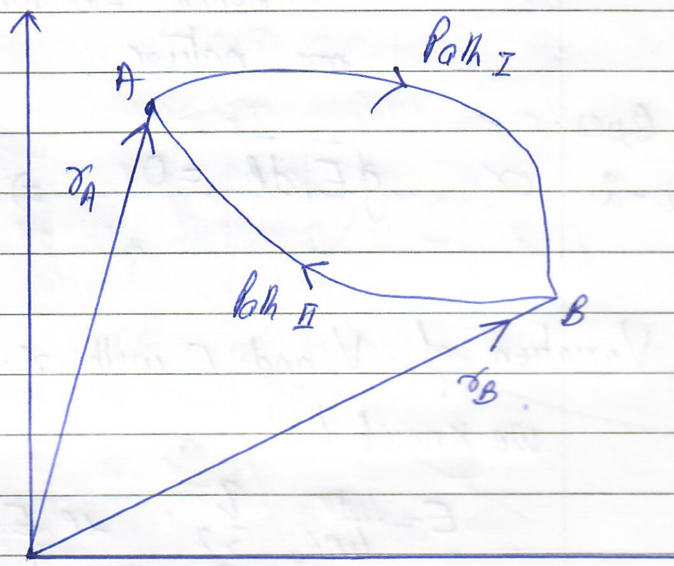
ie line integral of EFI from one pt to other pt is independent of the shape of the line b/w A & B and depends only upon position vectors r_A & r_B of the points (ie initial position & final position).

Ques → Show that electrostatic force is conservative in nat

or

Show that line integral of EFI over a closed pa is zero.



Ans → Electrostatic force is said to be conservative in nature, if work done in moving a charge against the electrostatic force (is independent of the path followed.) over a closed path is zero.

We know that WD in moving a unit +ve charge from point A to B along path I is

$$\frac{W_{AB}}{q_0} = -\int_{A \text{ path-I}}^{B} \vec{E}.\vec{dl} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \quad \text{—①}$$

Illy

$$\frac{W_{BA}}{q_0} = -\int_{B \text{ path-II}}^{A} \vec{E}.\vec{dl} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) \quad \text{—②}$$

Adding ① & ② we get.

~~Thus~~ $\cancel{\frac{W_{ABA}}{q_0} = \frac{W_{AB}}{q_0} + \frac{W_{BA}}{q_0}}$

$$\frac{W_{AB}}{q_0} + \frac{W_{BA}}{q_0} = -\int_{A \text{ path-I}}^{B} \vec{E}.\vec{dl} + \int_{B \text{ path-II}}^{A} \vec{E}.\vec{dl} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A} + \frac{1}{r_A} - \frac{1}{r}\right.$$

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$$\Rightarrow \boxed{\frac{W_{ABA}}{q_0} = -\oint \vec{E}\,\vec{dl} = 0}$$

for Ques no - 1 ie ~~W.D on~~ $\frac{W_{ABA}}{q_0} = 0$ $\Rightarrow$ W.D in moving a unit + charge against the electrostatic force over the closed path is equal to zero.

or $W_{ABA} = 0$

Hence electrostatic force is conservative ~~in~~ nature.

for Ques no - 2 or $\oint \vec{E}.\vec{dl} = 0$ $\Rightarrow$ line integral of EFI over closed path is zero.

Variation of V and E with r. ↦

we know that

$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$ ie $E \propto \frac{1}{r^2}$

and $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$ ie $V \propto \frac{1}{r}$

$\therefore$ for $r < 1$

the value of E is more than V

where as for $r > 1$

$E < V$

and for $r = 1$

$E = V$.

e.g for $r = \frac{1}{2}$ $E = \frac{1}{4\pi\epsilon_0} \frac{q}{(1/2)^2} = 4 \times \frac{1}{4\pi\epsilon_0} \times q$

w/a $V = \frac{1}{4\pi\epsilon_0} \frac{q}{1/2} = 2 \times \frac{1}{4\pi\epsilon_0} q$

$\therefore E > V$.

for $r > 1$ ie $r = 2$.

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{2^2} = \frac{1}{4\pi\epsilon_0} \times \frac{q}{4}$$

and $V = \frac{1}{4\pi\epsilon_0}\frac{q}{2}$

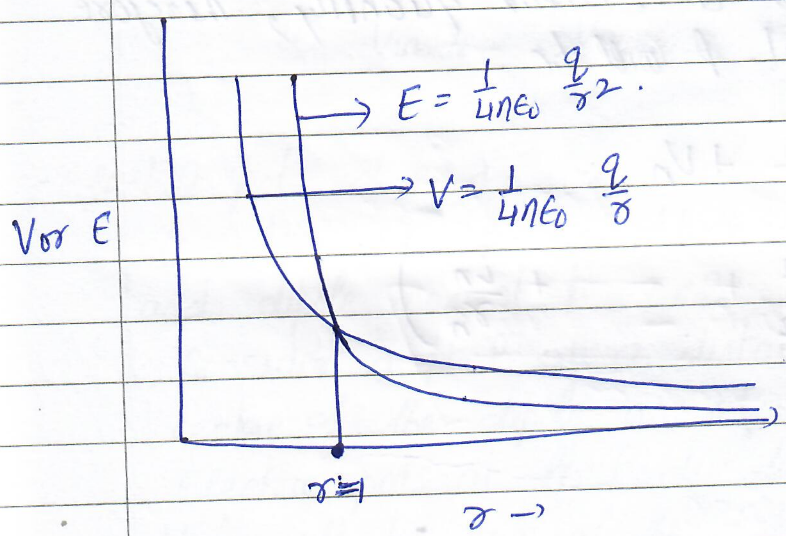
ie $V > E$

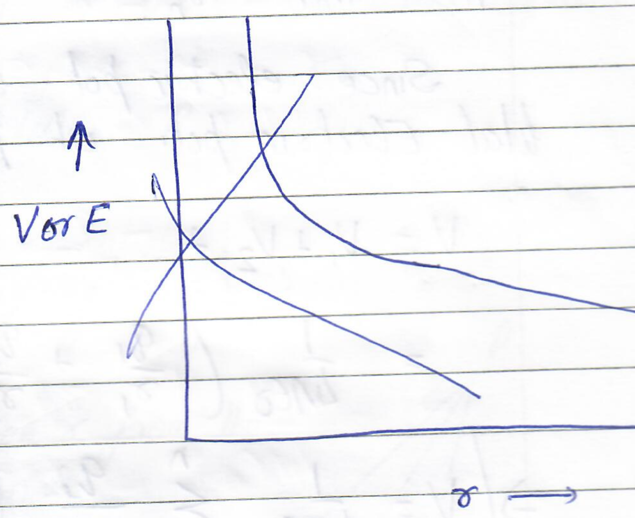
for $r = 1$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{1^2} = \frac{1}{4\pi\epsilon_0} q$$

$$V = \frac{1}{4\pi\epsilon}\frac{q}{1} = \frac{1}{4\pi\epsilon_0} q$$

ie $E = V$.



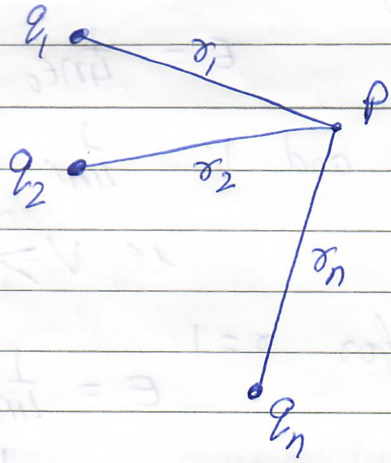


Electrostatic potential due to a charge distribution ↦

Total electric pot. at a point due to a charge distribution is equal to the algebraic sum of all the pot. at that point due to all the charges present in charge distribution.

Consider a charge distribution having n charges $q_1, q_2, ---q_n$ at a distance $r_1, r_2, ---r_n$ resp. from point P as shown.

Thus electric pot. at point P due to charge q_1 is.



$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1}$$

llly $$V_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2}$$

$$\vdots$$

$$V_n = \frac{1}{4\pi\epsilon_0}\frac{q_n}{r_n}.$$

Since electric pot. is a scalar quantity, the total Electric pot. at pt. P will be

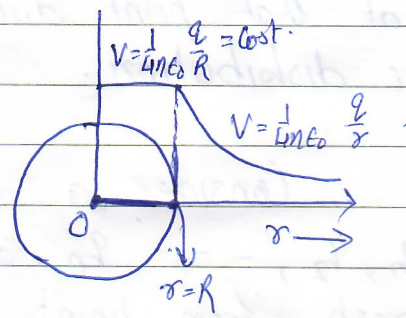
$$V = V_1 + V_2 + \text{——} + V_n$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \text{——} + \frac{q_n}{r_n}\right)$$

$$\Rightarrow \boxed{V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}}$$

Electric pot. due to a uniformly charged thin sph. shell ↦ .

Consider a uniformly charged thin sph. shell rad. R carrying charge q.



(a) when point P lies outside the shell ↦ then shell behaves as a pt. charge with entire charge supposed to be consentrated at its centre

$$\therefore V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

ie $V \propto \frac{1}{r}$

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~~ie pot decreases with increase in distance~~

(b) When pt. P lies on the surface of shell ↦

ie $r = R$.

$\therefore V = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$.

(C.) When pt. P lies inside the shell ie $r < R$

Since EFI at any pt. P inside the shell is equal to zero therefor pot. at any pt. P inside the shell will be const. ($\because E = -\frac{dV}{dr}$ and as $E = 0$ $\therefore V = $ Const)

and is equal to its value to that on its surface.

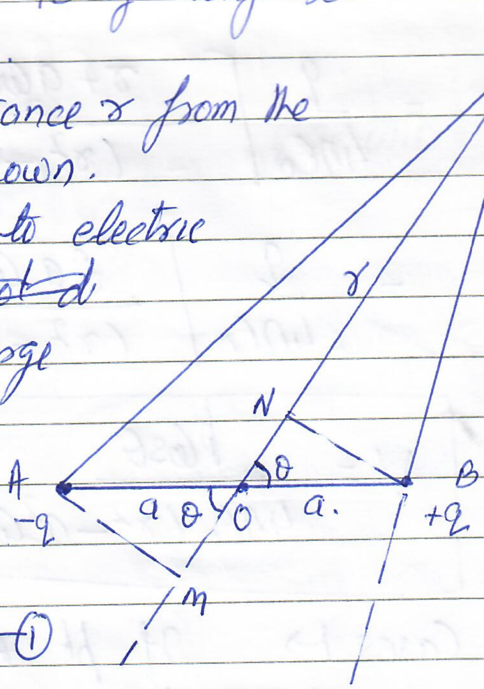
ie $V_{inside} = V_{surface} = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$.

Electric pot. due to an electric dipole ↦

Consider an electric dipole AB of length 2a and dipole moment $p = q \times 2a$.

Consider a pt P at a distance r from the centre of the dipole as shown.

Electric pot at Pt. P due to electric dipole will be sum of the ~~pot d~~ potentials due to +q charge and -q charge.



ie $V = V_A + V_B$

$V = \frac{1}{4\pi\epsilon_0} \frac{-q}{AP} + \frac{1}{4\pi\epsilon_0} \frac{q}{BP}$ —①

Now from rt angle Δ AMO

$\frac{OM}{AO} = \cos\theta$

$\Rightarrow OM = AO\cos\theta$

$\Rightarrow OM = a\cos\theta$.

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Since length of the dipole is very small

$\therefore AP \cong MP$

$\Rightarrow AP = OP + OM$

$AP = r + a\cos\theta$ — ②

Ily from rt $\angle$ ~~tran~~ triangle ONP

$\frac{ON}{OB} = \cos\theta$

$\Rightarrow ON = a\cos\theta$

$\therefore BP \cong PN$

$\Rightarrow BP = OP - ON$

$BP = r - a\cos\theta$ — ②

substitute ② & ③ in ① we get

$$V = \frac{1}{4\pi\epsilon_0}\frac{-q}{(r+a\cos\theta)} + \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a\cos\theta)}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r+a\cos\theta - r + a\cos\theta}{(r^2 - a^2\cos^2\theta)}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{2a\cos\theta}{(r^2-a^2\cos^2\theta)}\right]$$

$$\Rightarrow \boxed{V = \frac{p\cos\theta}{4\pi\epsilon_0(r^2-a^2\cos^2\theta)}}$$

Special Cases $\mapsto$ If pt P lies on the axial line

then $\theta = 0^\circ$.

$$\boxed{\therefore V_{axial} = \frac{p}{4\pi\epsilon_0(r^2-a^2)}}$$

If $r >> a$

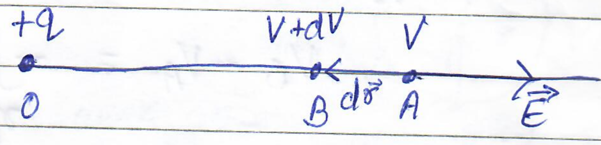
then $V_{axial} = \frac{p}{4\pi\epsilon_0 r^2}$

(iii) If point P lies on the equitorial line then $\theta = 90°$

$\therefore V_{equi} = 0$ $\{\because \cos 90 = 0$

Relation b/w EFI and pot. gradient ↦

Consider a charge +q at origin O. Let A and B are two points separated by a very small distance $d\vec{r}$.



Work done in moving the test charge q_0 from A to B against the electric field $\vec{E}$ is

$$W = \vec{F}\cdot d\vec{r} = q_0\vec{E}\cdot d\vec{r} = q_0 E\, dr \cos 180$$

$$\Rightarrow W = -q_0 E\, dr \quad \text{———} (1)$$

Also

$$\frac{W}{q_0} = V_B - V_A$$

$$W = q_0(V_B - V_A) \qquad \{\text{ie W.D} = \text{charge} \times$$

$$\Rightarrow W = q_0\, dV \quad \text{———} (2)$$

from (1) & (2)

$$-q_0 E\, dr = q_0\, dV$$

$$\Rightarrow \boxed{E = -\frac{dV}{dr}}$$

ie EFI = − Pot. gradient.

Thus EFI at any point is equal to −ve of pot. gradient at that point.

Also −ve sign shows that direction of EFI is in the direction of decreasing pot.

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Computing electric pot. from EFI ↦ If EFI at a
is known then electric pot can be calcula

We know that

$$\frac{W_{AB}}{q_0} = -\int \vec{E}\cdot d\vec{r} = V_B - V_A$$

$$\therefore V_B - V_A = -\int_{r_A}^{r_B} \vec{E}\cdot d\vec{r}$$

if pt A is at ∞ then $V_A = 0$

~~then~~ $\therefore V_B = -\int_{\infty}^{r} \vec{E}\cdot d\vec{r}$ { where

Alternative method. ↦ w.k.T

$$E = -\frac{dV}{dr}$$

$$\therefore dV = -E\,dr$$

$$\Rightarrow \int_{V_1}^{V_2} dV = -\int_{r_1}^{r_2} E\,dr$$

$$\Rightarrow \boxed{V_2 - V_1 = -\int_{r_1}^{r_2} E\,dr}$$

if pt A is at ∞

ie $r_1 = \infty$ and $r_2 = r$ then $V_A = 0$ and $V_2 = V$

$$\therefore V = -\int_{\infty}^{r} E\,dr$$

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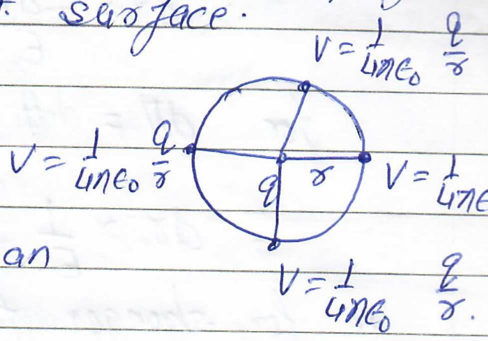
Equipotential Surface ↦ Any surface that has same po[tential] at each and every point on it is called equip[otential] surface. eg. surface of the sphere carrying ch[arge] q at its centre is equipot. surface.



Properties ↦

① W.D. in moving a charge over an equipot. surface is zero.

W.K.T $\frac{W_{AB}}{q_0} = V_B - V_A$

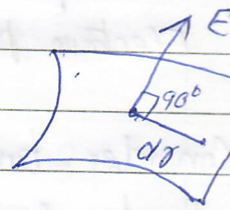
for an equipotential surface

$$V_A = V_B$$

$\therefore \frac{W_{AB}}{q_0} = 0 \Rightarrow W_{AB} = 0.$

② EFI at any point on the equipot. surface is [a]
normal to the surface.

W.K.T $dV = \vec{E}.d\vec{r}$



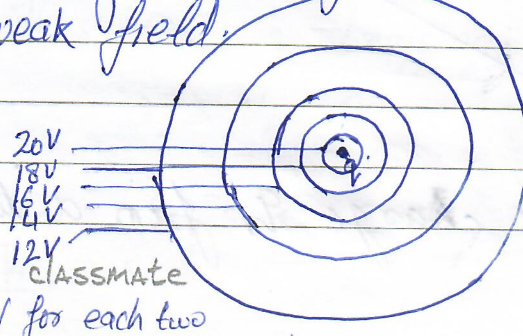
since for an equipot. surface

$$dV = 0$$

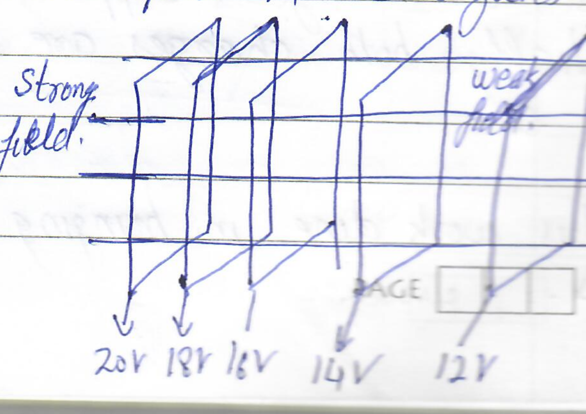
$\therefore \vec{E}.d\vec{r} = 0 \Rightarrow E\,dr\cos\theta = 0$

$\Rightarrow \vec{E}$ and $d\vec{r}$ are ⊥

③ Equipot. surfaces are closer together in the region of strong electric field and farther apart in the regions [of] weak field.



ie dV for each two surface is 2V or const.



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we know that

$$E = -\frac{dV}{dr}$$

$$\therefore dr = -\frac{dV}{E}$$

for dV = diff. in pot b/w two equipot surface = Const.

$$dr \propto \frac{1}{E}$$

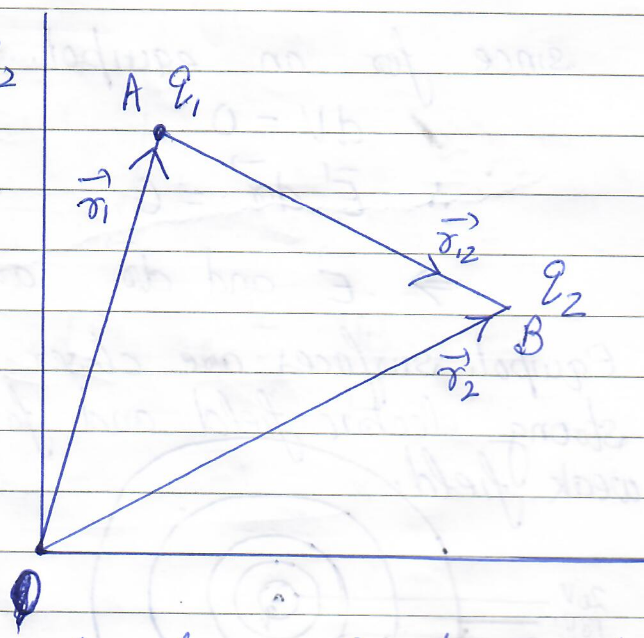
ie for stronger field dr (Separation b/w two surf is small and vice versa.

Note ↦ if EFI is uniform then equipotential surfa will be equidistant.

Electric potential Energy ↦ Electric potential energy to a system of point charges may be d/a the workdone in bringing all the charges from ∞ to respective locations.

(a) Electric pot. Energy due to asystem of two point charg

Consider two charges q_1 & q_2 at points A & B having PV $\vec{r_1}$ & $\vec{r_2}$ resp.



In order to find the Electric pot. Energy due to these two charges, suppose, initially both charges are at ∞.

Thus work done in bringing the charge q_1 from ∞ to is.

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$\frac{W_{AB}}{q_0} = (V_B - V_A)$

$W_1 = V_A \times q_1$ {∵ W.D = charge × Pot.

$\Rightarrow W_1 = 0$ —— ① { ∵ $V_A = 0$ since both the charges were at ∞ and q_1 can't exert pot. (or force) on itself. }

Now WD in bringing the charge q_2 from ∞ to B against the pot of charge q_1 is.

$W_2 = V_B \times q_2$.

$= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{12}} \times q_2$

$W_2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}$. —— ②

Thus total WD and ~~the~~ hence the electric pot. Energy is

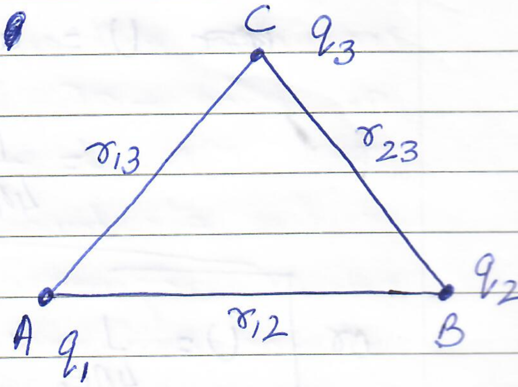
$U = W_1 + W_2$

$$\boxed{U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}}$$ {using ① & ②}

(b) Electric pot. energy due to system of three charges

Consider three charges q_1, q_2, q_3 at point A, B & C resp.

In order to find Electric pot. Energy due to these three charges, suppose, initially all three charges are at ∞



Thus WD in bringing charge q_1 from ∞ to A is

$W_1 = V_A \times q_1$

$\Rightarrow W_1 = 0$ —— ① { ∵ $V_A = 0$ }.

Ily. W.D. in bringing charge q_2 from ∞ to B against the pot. of charge q_1 at pt B is.

$$\omega_2 = V_B \times q_2$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{12}} \times q_2$$

$$\Rightarrow \omega_2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}} \quad \text{——— (2)}$$

Ily W.D in bringing charge q_3 from ∞ to C against the pot. of charge q_1 and q_2 at pt C is

$$\omega_3 = V_C \times q_3$$

$$= \left[\frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{13}} + \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_{23}}\right] \times q_3$$

$$\Rightarrow \omega_3 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{r_{23}} \quad \text{——— (3)}$$

Thus total WD and hence the electric pot. Energy due to 3 charges is.

$$U = \omega_1 + \omega_2 + \omega_3$$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right] \quad \{\text{using (1),(2)}$$

or
$$\boxed{U = \frac{1}{4\pi\epsilon_0} \times \frac{1}{2} \sum_{i=1}^{3} \sum_{\substack{j=1 \\ i \neq j}}^{3} \frac{q_i q_j}{r_{ij}}}$$

or we can also write this as

$$U = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{3} \sum_{\substack{j=1 \\ i<j}}^{3} \frac{q_i q_j}{r_{ij}}.$$

Similarily for a system of n charges electric pot. energy can be written as

$$U = \frac{1}{4\pi\epsilon_0} \times \frac{1}{2} \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} \frac{q_i q_j}{r_{ij}}$$

or $$U = \frac{1}{4\pi\epsilon_0} \sum_{\substack{i=1 \\ i < j}}^{n} \sum_{j=1}^{n} \frac{q_i q_j}{r_{ij}}$$