

Electromagnetic Waves

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Question  $\rightarrow$  Show that Ampere's circuital law is inconsistent. What is modified Maxwell's Ampere's circuital law. Show that modified Maxwell's Ampere's circuital law is consistent.

Ans:  $\rightarrow$  Consider a parallel plate capacitor being charged by a battery. Let  $I$  is the current flowing through the wires at any instant. Consider two closed loop  $C_1$  and  $C_2$  as shown

On applying Ampere's Circuital law for the closed loop  $C_1$  we get.

$$\oint_{C_1} \vec{B} \cdot d\vec{\ell} = \mu_0 I \quad \text{--- (1)}$$

Similarly for closed loop  $C_2$ ,

$$I = 0$$

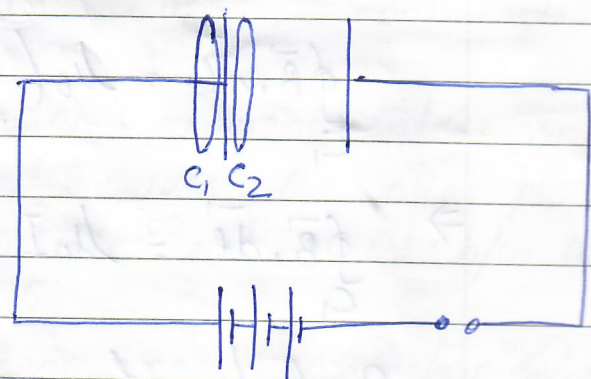
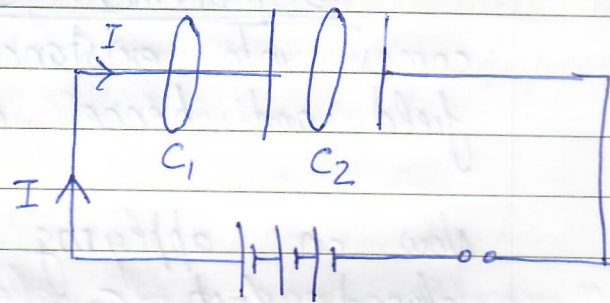
$$\therefore \oint_{C_2} \vec{B} \cdot d\vec{\ell} = \mu_0(0)$$

$$\Rightarrow \oint_{C_2} \vec{B} \cdot d\vec{\ell} = 0 \quad \text{--- (2)}$$

But if two loops are infinitesimally close to each other then

$$\oint_{C_1} \vec{B} \cdot d\vec{\ell} \text{ must be equal to } \oint_{C_2} \vec{B} \cdot d\vec{\ell}'$$

$$\text{i.e. } \oint_{C_1} \vec{B} \cdot d\vec{\ell} = \oint_{C_2} \vec{B} \cdot d\vec{\ell}' \quad \text{--- (3)}$$



But eqn (3) is not in accordance with no. (1) and (2). Thus we can say that circuital law is logically inconsistent.

Thus Maxwell's gave the modified law  
k/a Maxwell's Ampere's circuital law.  
to modified law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 (I + I_D)$$

where  $I_D = \epsilon_0 \frac{d\phi_E}{dt}$  = Displacement Current

Displacement Current is that current which comes into existence in a region where electric field changes and hence electric flux changes.

Now on applying Modified Ampere's law to closed loop  $C_1$  we have,

$$\oint_{C_1} \vec{B} \cdot d\vec{\ell} = \mu_0 (I + 0)$$

$$\Rightarrow \oint_{C_1} \vec{B} \cdot d\vec{\ell} = \mu_0 I \quad \text{--- (5)}$$

{  $\because I_D = 0$   
during the charging of capacitor and electric field only b/w plates }

Similarly for closed loop  $C_2$   
conduction current  $I = 0$

$$\therefore \oint_{C_2} \vec{B} \cdot d\vec{\ell} = \mu_0 (0 + I_D)$$

$$\Rightarrow \oint_{C_2} \vec{B} \cdot d\vec{\ell} = \mu_0 I_D = \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

If  $E$  is the value of electric field b/w the two plate of capacitor at any instant then

$$\oint_C \vec{B}' \cdot d\vec{\ell}' = \mu_0 \epsilon_0 \frac{d}{dt} EA \quad \left\{ \phi_E = EA \right\}$$

$$= \mu_0 \epsilon_0 \frac{d}{dt} \frac{Q}{A \epsilon_0} A \quad \left\{ \because E = \frac{Q}{A \epsilon_0} \right\}$$

$$= \mu_0 \epsilon_0 \cdot \frac{1}{\epsilon_0} \frac{dQ}{dt}$$

$$\Rightarrow \oint_C \vec{B}' \cdot d\vec{\ell}' = \mu_0 I \quad \text{--- (6)} \quad \left\{ \because I = \frac{dQ}{dt} \right\}$$

Eqn no. (5) and (6) are in close agreement with eqn no. (3). Thus modified Maxwell's Ampere's circuital law is logically consistent.

**Ques H** Comment on the statement that conduction and displacement currents are individually discontinuous, but their sum is continuous.

**Ans H** We know that conduction current enters one plate of the capacitor and leaves the other plate i.e. conduction current does not exist b/w the two plates of the capacitor.

where as since there is no electric field outside the plates of the capacitor. Therefore displacement current does not exist outside the two plate and exist only b/w the two plates due to change in electric field and hence change in electric flux.

i.e. between the plates conduction current

and displacement current

$$I_D = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt} EA = \epsilon_0 \frac{dQ}{dt}$$

$$\Rightarrow I_D = \frac{dQ}{dt}$$

Thus sum,  $I_C + I_D = 0 + \frac{dQ}{dt} =$

where as outside the plates conduction exists

ie  $I_C = \frac{dQ}{dt}$

w/a  $I_D = \epsilon_0 \frac{d\phi_E}{dt} = 0$

$\therefore$  Sum  $I_C + I_D = \frac{dQ}{dt} + 0 = I$

ie individually  $I_C$  and  $I_D$  are  
w/a sum is continuous

### Maxwell's eqns of electromagnetism $\Rightarrow$

Basic principles of electromagnetism known as Maxwell's eqns of electromagnetism

① Gauss's law of electrostatics  $\Rightarrow$

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon_0}$$

2. Gauss's law of magnetism  $\Rightarrow$

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

③ Faraday's law of electromagnetism  $\Rightarrow$

we know that

$$\oint \vec{E} = -\frac{d\phi_m}{dt}$$

$$\Rightarrow \oint_L \vec{E} \cdot d\vec{\ell} = - \frac{d}{dt} \int \vec{B} \cdot d\vec{s}$$

$$\left\{ \begin{aligned} \because e = v_B - v_A = \omega \\ = \oint_L \vec{E} \cdot d\vec{\ell} \end{aligned} \right.$$

and  $\phi_m = \int \vec{B} \cdot d\vec{s}$

4. Modified Maxwell's Ampere's Law  $\rightarrow$

$$\oint_L \vec{B} \cdot d\vec{\ell} = \mu_0 (I + I_D)$$

$$\Rightarrow \oint_L \vec{B} \cdot d\vec{\ell} = \mu_0 \left( I + \epsilon_0 \frac{d\phi_E}{dt} \right)$$

$$\Rightarrow \oint_L \vec{B} \cdot d\vec{\ell} = \mu_0 \left( I + \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{s} \right)$$

### Electromagnetic waves $\rightarrow$

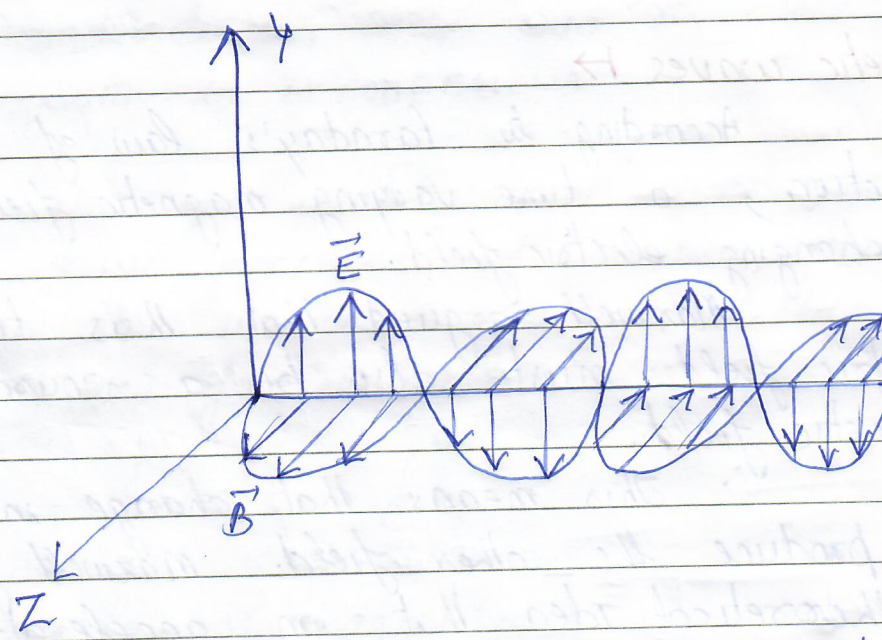
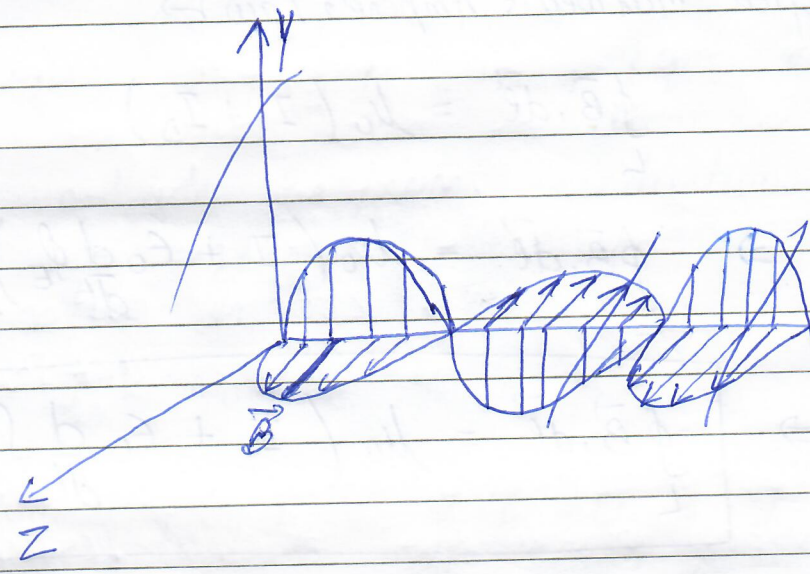
According to Faraday's law of electromagnetic induction, a time varying magnetic field is a source of changing electric field.

Maxwell argued that thus time varying electric field must also be a source of changing magnetic field.

This means that change in either field will produce the other field. Maxwell further gave the theoretical idea that an accelerated charged particle produces sinusoidal magnetic field, which in turn produces sinusoidal electric field. These two mutually perpendicular electric and magnetic fields produce electromagnetic waves in the direction perpendicular to both the fields.

(e.g. if E-field varies along Y axis  
 then em wave will propagate along X axis.)

Thus whenever electric and magnetic fields vary sinusoidally and are perpendicular to each other, we get em. waves in a direction perpendicular to both the fields.



Variation in electric and magnetic fields is in same phase. i.e. both attain their maxima and minima at the same instant and at the same place (x).

Also velocity of em wave is

classmate 
$$c = \frac{E}{B} \quad \text{or} \quad c = \frac{E_0}{B_0} \quad \text{or} \quad c = \dots$$

**Source of em waves**  $\rightarrow$  A stationary charged particle produces only electric field. where as a charge moving with uniform velocity can produce mag. field. But this mag. field does not change with time. Thus neither a stationary charge, nor a charge moving with uniform velocity can produce em waves.

Maxwell pointed out that an accelerated charged ~~pro~~ particle produces time varying magnetic field and hence time varying electric field. The accelerated charged particle produces em wave.

eg. in LC oscillations a charge oscillates harmonically with time. i.e. charge ~~is~~ is an accelerating charge and hence produce em waves.

Whenever an electron jumps from higher orbit to lower orbit, its velocity changes hence produce em waves.

**History of em waves**  $\rightarrow$  In 1865 Maxwell gave a theoretical idea of em. waves. He said an accelerated charge produces time varying magnetic field which in turn produces time varying electric field. two fields are mutually ~~are~~ are perpendicular to each other and hence produce em waves in a direction  $\perp$  to both the fields.

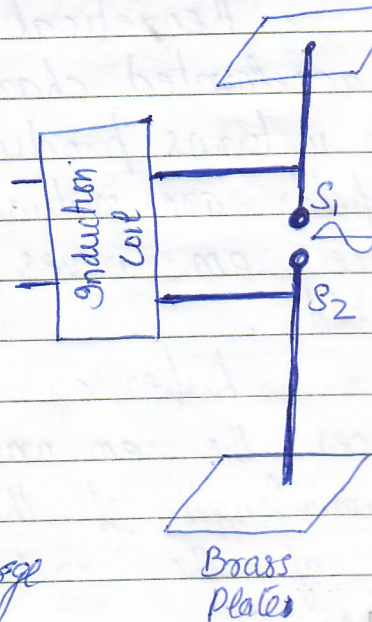
Later on in 1887 Hertz experimentally produces the em waves by using spark oscillator. the wavelength of these waves was very high and was of the order of 6m.

In 1894. Indian scientist J.C. Bose produced em wave of comparatively wave length ~~and hence~~ of high energy in range of 5mm to 25mm.

In 1896 Marconi ~~was~~ become able to transmit em waves over the long distance making use of antenna.

Marconi first established wire communication across the English channel with 50kms.

**Hertz Expt**  $\rightarrow$  Hertz used spark oscillator to produce em waves. Hertz experimental arrangement consist of two large square brass plates of length 40cm and held at a distance of 60cm from each other. These two plates are connected to two highly polished brass spheres  $S_1$  and  $S_2$  with the help of thick copper wire. The separation b/w the two spheres is 3 cms. Thick copper wire are attached to an induction coil as shown. Induction coil sets up a very high potential diff. across  $S_1$  and  $S_2$ . This high pot. diff. ionizes the air b/w the spheres and makes the gap conducting. The ions and electrons so produced oscillates back and forth. Thus an oscillatory discharge is produced.





of the plates occurs through the conducting gap, resulting in production of em waves.

The electromagnetic waves so produced are detected noticing the spark produced b/w two spheres.

In Hertz experiment metal plates form a capacitor of low capacitance  $C$  and connecting wire offers a low inductance  $L$ . Thus em waves of high frequency

$$\nu = \frac{1}{2\pi\sqrt{LC}} \text{ are generated.}$$

Transverse Nature of em waves  $\rightarrow$

Consider a plane electromagnetic wave front propagating along  $x$  axis.

Let the plane wave front ~~be~~.

Let ABCD is the position of plane wave front after time  $t$ . Since wave is propagating along  $x$  axis therefore the value of electric field  $E$  and magnetic field  $B$  will depend only upon the  $x$  co-ordinates.

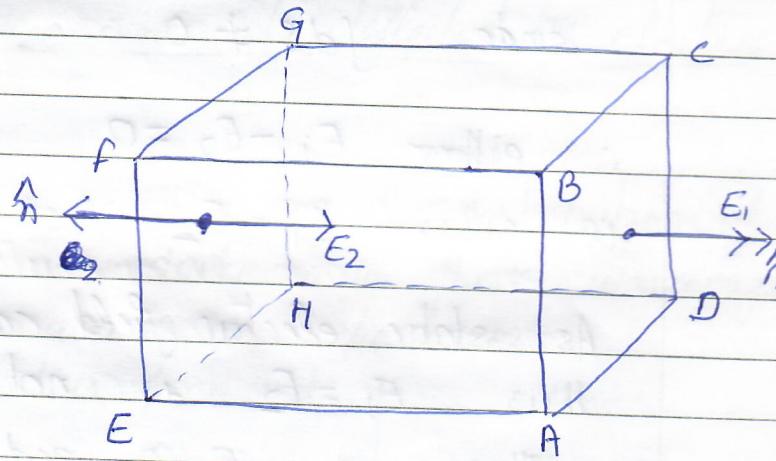
Since closed surface ABCDEFGH does not enclose any charge, therefore from Gauss's law,

$$\oint \vec{E} \cdot d\vec{s} = 0$$

ABCEFGH

$$\Rightarrow \int_{ABCD} \vec{E} \cdot d\vec{s} + \int_{EFGH} \vec{E} \cdot d\vec{s} + \int_{EADH} \vec{E} \cdot d\vec{s} + \int_{FBCG} \vec{E} \cdot d\vec{s} + \int_{EABF} \vec{E} \cdot d\vec{s} + \int_{HDCG} \vec{E} \cdot d\vec{s} = 0$$

Since value of  $E$  does not depend upon  $y$  and  $z$  coordinates.  $\int \vec{E} \cdot d\vec{s}$  for the faces normal to  $y$  and  $z$  axis will cancel out in pairs.



$$\Rightarrow \int_{ABCD} \vec{E}_1 \cdot d\vec{s} + \int_{EFGH} \vec{E}_2 \cdot d\vec{s} = 0$$

Let the value of  $E$  is  $E_1$  for face ABCD face EFGH. Thus we get.

$$\int_{ABCD} E_1 ds \cos 0 + \int_{EFGH} E_2 ds \cos 180 = 0$$

$$\Rightarrow \int_{ABCD} E_1 ds - \int_{EFGH} E_2 ds = 0$$

$$\Rightarrow (E_1 - E_2) \int ds = 0 \quad \left\{ \because \int ds \neq 0 \right.$$

since  $\int ds \neq 0$

$$\therefore \text{either } E_1 - E_2 = 0$$

ie either  $E_1 = E_2$  or  $E_1 = 0$  and

As static electric field can't produce em thus  $E_1 = E_2$  is not possible

Thus we  $E_1 = 0$  and  $E_2 = 0$  ie the no electric field along x axis or in Electric field is  $\perp$  to x axis,  $\perp$  to  $\perp$  to the direction of propagation of a

Similarly by using Gauss's law magnetism ie  $\oint_{ABCDEFGH} \vec{B} \cdot d\vec{s} = 0$ , we can prove

magnetic field is  $\perp$  to the direction of propagation of wave.

Thus em waves are transverse

## Properties of em waves $\rightarrow$

- ① Electromagnetic waves are produced by accelerated charge.
- ② em waves do not require any material medium for their propagation.
- ③ Electric field and magnetic field are  $\perp$  to each other as well as direction of propagation of waves
- ④ Velocity of em waves is equal to velocity of light  

$$c = \frac{E}{B} \quad \text{or} \quad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ ms}^{-1}$$
- ⑤ Electromagnetic waves are not deflected by electric & magnetic fields.

Electromagnetic spectrum  $\rightarrow$  The orderly distribution of different em wave <sup>having different properties</sup> in accordance with their wavelength or frequencies is called em spectrum.

S.No.	Name	Wavelength Range	How it is produced	uses.
1.	γ-Rays	$< 10^{-3} \text{ nm}$ $< \frac{1}{100} \text{ \AA}$	Radioactive decay of the nucleus.	<ol style="list-style-type: none"> <li>① In radiotherapy for treatment of cancer</li> <li>② To study the structure of atomic nucleus</li> </ol>
2.	X-Rays	<del><math>1 \text{ nm}</math> to <math>10^{-3} \text{ nm}</math></del> <del><math>10^{-3} \text{ nm}</math> to <math>10 \text{ \AA}</math></del> $\frac{1}{100} \text{ \AA}$ to $10 \text{ \AA}$	when a fast moving $e^-$ beam is allowed to fall on a heavy metal	<ol style="list-style-type: none"> <li>① In medical diagnosis</li> <li>② to ionize the gas</li> <li>③ for photoelectric emission</li> </ol>
3.	U.V. Rays	<del><math>4000 \text{ \AA}</math> to <math>10 \text{ \AA}</math></del> $10 \text{ \AA}$ to $4000 \text{ \AA}$	when an electron jumps from higher orbit to lower orbit.	<ol style="list-style-type: none"> <li>① In food preservation</li> <li>② To detect the forged documents or currency</li> <li>③ To reveal finger prints</li> </ol>

$\frac{1}{1000}$   $10^{-10} m$   
 $10^{10} A$   $1 m$



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S.No	Name	Wavelength Range	How it is produced	Uses
4.	Visible light	$4000 \text{ \AA}$ to $7000 \text{ \AA}$	When an electron jumps from higher orbit to lower orbit	<ol style="list-style-type: none"> <li>To see around</li> <li>to cause</li> </ol>
5.	Infrared rays	$7000 \text{ \AA}$ to $1 \text{ mm}$	—do—	<ol style="list-style-type: none"> <li>In rem</li> <li>In green</li> </ol>
6.	Microwaves	$1 \text{ mm}$ to $100 \text{ mm}$	In vacuum Tubes.	<ol style="list-style-type: none"> <li>In RA</li> <li>In mic</li> <li>In long telecom</li> </ol>
7.	Radowaves	$> 100 \text{ mm}$	In L-C oscillatory circuits	<ol style="list-style-type: none"> <li>In ra telecom</li> <li>Wireless</li> </ol>

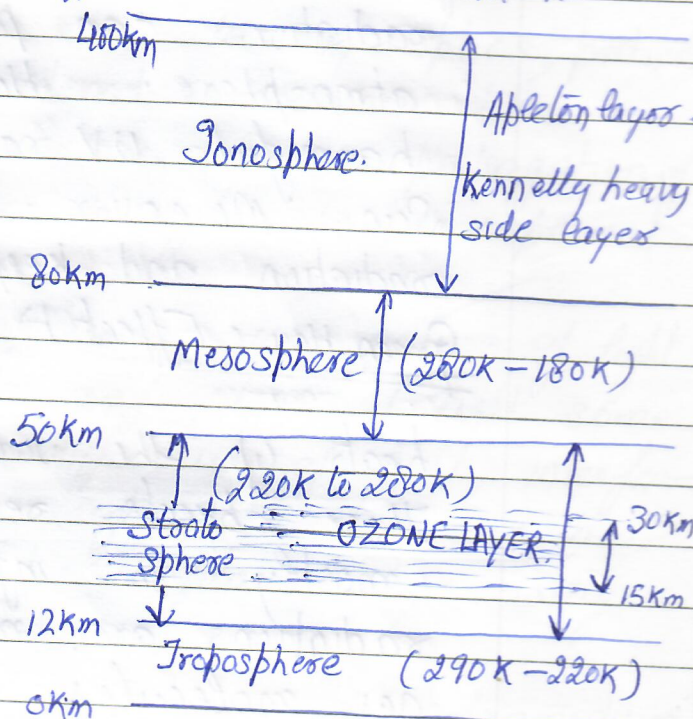
Electromagnetic waves in the decreasing order of energy and hence frequency:—

$\gamma$ -Rays  $>$  X Rays  $>$  UV Rays  $>$  Visible  $>$  I.R rays  
 Highest freq.  
 $\therefore$  Highest energy

Earth's atmosphere  $\rightarrow$  The thick envelop of air the earth is called earth's atmosphere. upto  $400 \text{ km}$  above the surface of earth atmosphere, can be divided in four

① **Troposphere** → It extends upto 12km from the surface of earth. Temperature falls with height from 290K to 220K. This layer is responsible for all the weather phenomena that affect our environment.

② **Stratosphere** → It extends from 12kms to 50kms. This portion of the atmosphere also contains ozone layer which protects us from harmful UV radiations. The temp. of stratosphere rises from 220K to 280K



③ **Mesosphere** → It extends from 50km to 80kms. The temp. of this region falls from 280K to 180K.

④ **Ionosphere** → This layer extends upto from 80km to 400km. Its temp. increases with height from 180K to 200K. In this layer most of the gases are in ionized form. This ionization is caused by high energy cosmic rays, U.V rays and X rays coming from the sun. In this region beyond 110km concentration of electrons suddenly increases and extends upto few kms. This layer of electrons of high electron density is called Kennelly heavy side layer. Beyond this layer electron density decreases with height and at about 250kms above the surface of earth another layer of electrons classmate known as Appleton layer is met.

Radiowaves  
Lowest freq.  
Lowest energy.

## Behaviours of earth's atmosphere towards $\leftarrow$ em

Earth's atmosphere is transparent to most of the em radiations.  $\leftarrow$  most of radiations can pass through the earth's atmosphere. However ozone layer blocks harmful UV radiations ~~com~~ coming from Sun. Moreover it also traps the outgoing radiation and keeps the earth warm. Green House Effect is

The radiation from the sun heats up the various objects on earth. These objects re-radiate the radiation mostly in infra red region. The radiations are mostly reflected back by gas molecules and clouds lying in lower atmosphere. Thus earth's atmosphere becomes richer in infra red radiation causing Green House Effect which makes the earth's surface warm.