

E

The study of electric charges in motion is called current electricity.

Electric current $\rightarrow$ The flow of electric charges through a conductor constitutes an electric current.

Mathematically, the rate of flow of electric charge through a conductor is called electric current.

If q charge flows through a conductor in time t , then

$$I = \frac{q}{t}$$

But if flow of charge is not uniform then dq charge flows through a conductor in a very small time dt then instantaneous value of current is

$$I = \frac{dq}{dt}$$

Units $\rightarrow$

$$I = \frac{q}{t} = \frac{C}{s} = Cs^{-1} = \text{Ampere (A)}$$

If $q = 1C$ and $t = 1s$

then $I = \frac{1C}{1s} = 1A$

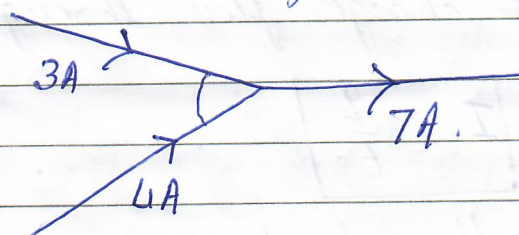
Thus current is said to be 1 ampere ~~1 ampere~~ if 1 coulomb of charge flows through a conductor in 1 second.

By convention, the direction of motion of +ve charge is taken as the direction of current.

However direction of flow of electrons gives the direction of electronic current.

Electric current is a scalar quantity; $\rightarrow$ Although current has both magnitude and direction, yet it is a scalar quantity, because currents are added up algebraically and not vectorially.

eg. ~~2~~ wires carrying currents 3A and 4A always ~~produce~~ add up to give 7A current, whatever be the angle b/w the wires.



Charge / Current carriers $\rightarrow$

1. In solids $\rightarrow$ free electrons are the charge carriers in solids.
2. In Liquids/Electrolytes $\rightarrow$ In liquids current is due to movement of ions.
3. In gases $\rightarrow$ In gases current flows due to movement of ions and electrons produced due to ionization of gases.

Drift Velocity $\rightarrow$ The average velocity with which free electrons get drifted towards the +ve terminal of the battery is called drift velocity.

Expression for drift velocity $\rightarrow$

In the absence of any external electric field, free electrons move inside the conductor randomly with a very large thermal velocity. But since motion of these electrons is random, therefore their average velocity is zero.

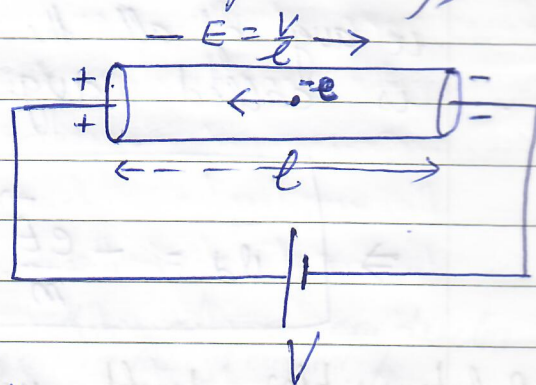
ie
$$\vec{u}_1 + \vec{u}_2 + \dots + \vec{u}_n = 0.$$

classmate PAGE

--	--	--	--	--

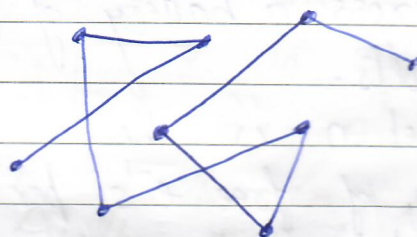
But when external electric field $\vec{E}$ is applied by applying the pot. diff. V , then all electrons get accelerated towards the +ve terminal of battery, which can be given as

$$\vec{a} = \frac{\vec{F}}{m} = -\frac{e\vec{E}}{m} \quad \text{--- (2)}$$



But this acceleration acts on the electron for a very small time. Because while travelling towards the +ve terminal, it suffers frequent collisions with the ions or atoms or electrons of the conductor.

The average time b/w two successive collisions of an electron is called relaxation time.



If $\tau_1, \tau_2, \dots, \tau_n$ are the relaxation times for n free electrons resp. the velocity of the first electron can be given as

$$\vec{v}_1 = \vec{u}_1 + \vec{a} \tau_1$$

Similarly $\vec{v}_2 = \vec{u}_2 + \vec{a} \tau_2$

$$\vdots$$

Similarly $\vec{v}_n = \vec{u}_n + \vec{a} \tau_n$

Thus from the definition of drift velocity

$$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \dots + \vec{v}_n}{n}$$

$$\text{classmate} = \frac{\vec{u}_1 + \vec{u}_2 + \dots + \vec{u}_n}{n} + \vec{a} (\tau_1 + \tau_2 + \dots + \tau_n)$$

$$\Rightarrow \vec{v}_d = 0 + \vec{a} \tau$$

{ " using ① }

where $\tau = \frac{\tau_1 + \tau_2 + \dots + \tau_n}{n} = \text{avg. relaxation time}$

i.e. avg. of all the relaxation times of free electrons is called avg. relaxation time.

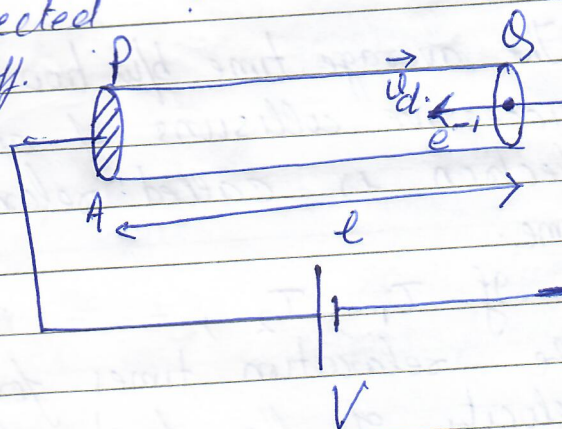
$$\Rightarrow \boxed{\vec{v}_d = -\frac{eE}{m} \tau}$$

{ using ② }

Relation b/w drift velocity and electric current

Consider a conductor of length l and area cross-section A is connected across a battery of pot. diff. V volts.

Let n is the no. of free e^- 's per unit volume of the conductor. Thus



Total free electrons in the conductor, $N = nAl$

Thus total charge in the conductor due to these free electrons is

$$q = Ne$$

(mag. only)

$$\Rightarrow q = nAle \quad \text{--- ①}$$

This entire charge will flow through the wire when free e^- at end Q will reach at end P. Thus time taken for the entire charge q to pass is

$$t = \frac{l}{v_d} \quad \text{--- ②}$$

Thus current flowing through the conductor

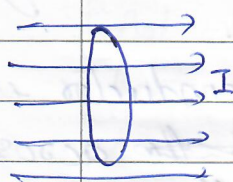
$$I = \frac{\text{Total charge flowing}}{\text{Time taken}}$$

$$I = \frac{n A l e}{t/d_d.}$$

$$\boxed{I = n e A v_d.}$$

Current Density $\Rightarrow$

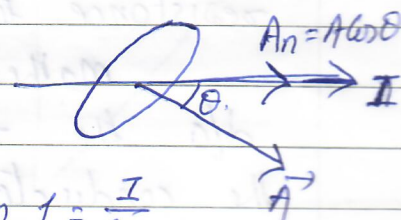
Current flowing through a conductor per unit area A is called current density.



$$j = \frac{I}{A} = \frac{n e A v_d}{A}$$

$$\Rightarrow \boxed{j = n e v_d.}$$

$$\text{or } \vec{j} = n e \vec{v}_d$$



$$\text{then } j = \frac{I}{A_n} = \frac{I}{A \cos \theta}$$

$$\Rightarrow I = j A \cos \theta = j \cdot \vec{A}$$

Electron mobility $\Rightarrow$

The mobility of the free e^- 's is d/o the drift velocity acquired by the free e^- 's on applying unit electric field.

$$\text{ie } \mu = \frac{v_d}{E}$$

$$\text{since } v_d = \frac{e E \tau}{m} \quad (\text{mag. only})$$

$$\therefore \boxed{\mu = \frac{e \tau}{m}}$$

Ohm's Law $\Rightarrow$

According to Ohm's law, current flowing through a conductor is directly proportional to the pot. diff. applied across the ends of the conductor

$$\text{ie } V \propto I$$

$$\Rightarrow \boxed{V = R I}$$

classmate

where R is const. of proportionality and its value

depends upon the nature of the conductor, its length and area of cross-section.

Resistance of the conductor $\rightarrow$ is the obstruction possessed by the conductor to the flow of current.

Cause of resistance $\rightarrow$ When pot. diff. is applied across the conductor, free e^- s get accelerated towards the +ve terminal of battery. While moving towards the +ve terminal ~~their motion~~ they collide against the atoms and ions in the conductor and their motion gets obstructed causing resistance to the flow of charge/current.

Mathematically, resistance of the conductor is the ratio of the pot. diff. applied across the conductor to the value of current flowing through it.

$$R = \frac{V}{I}$$

Units

$$R = \frac{\text{Volt}}{\text{Ampere}} = \text{VA}^{-1} = \text{ohm } (\Omega)$$

if $V = 1 \text{ Volt}$ and $I = 1 \text{ Amp}$.

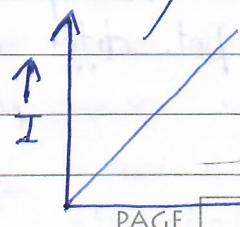
$$\text{then } R = \frac{1 \text{ Volt}}{1 \text{ Amp}} = 1 \text{ VA}^{-1} = 1 \Omega$$

Thus resistance of the conductor is said to be 1Ω if 1 Ampere of current flows through it on applying a pot. diff. of 1 volt.

Ohmic Conductors $\rightarrow$ The conductors which obey Ohm's law are called ohmic conductors.

For ohmic conductors, the graph b/w V and I is a straight line

classmate



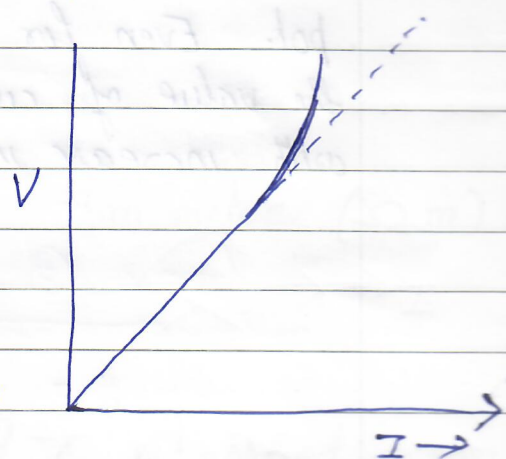
Non ohmic conductors or failure of Ohm's Law

Ohm's law is not a fundamental law in nature. There are many conductors which do not obey Ohm's law. Such conductors are called non ohmic conductors. For non ohmic conductors, graph b/w V and I is not a straight line.

Examples of Non-ohmic conductors

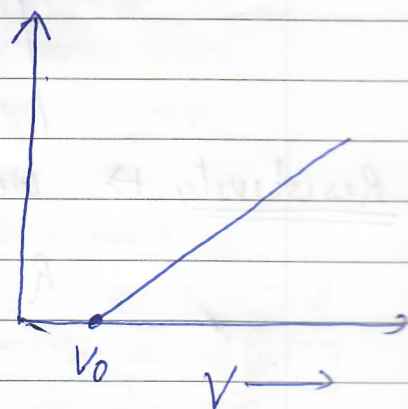
(1) Metallic conductors

For small value of current, metallic conductors obey Ohm's law. But when large current flows through them, due to heating effect of current their resistance increases and hence current flowing through the conductor does not remain proportional to the pot. diff. applied. Hence graph b/w V and I becomes non linear.



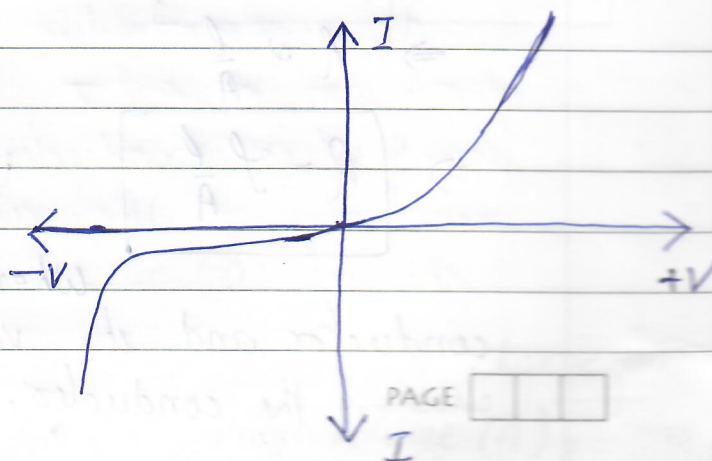
2. Water Voltameter:

In case of water voltameter no current flows through it, until applied pot. is more than the back emf V_0 . The cause of back emf is liberation of hydrogen at cathode and oxygen at anode.



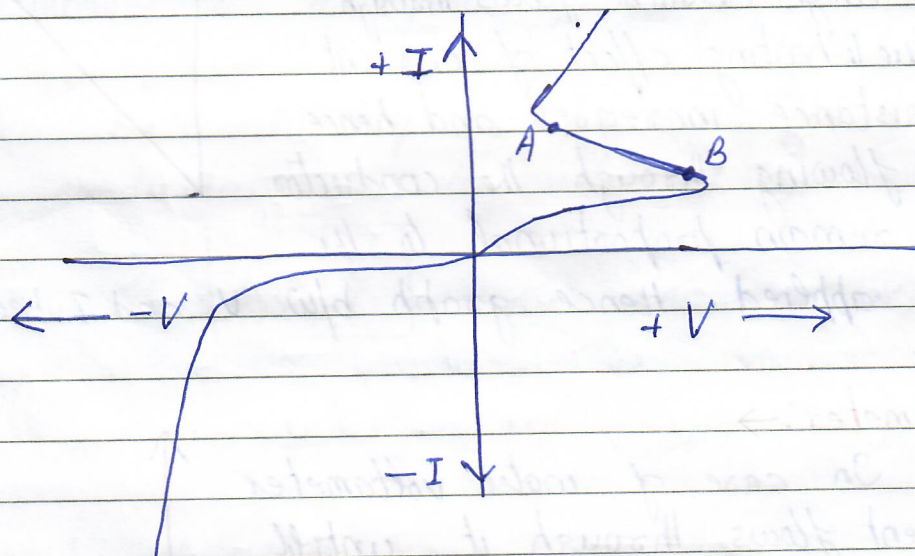
3 p-n junction diode:

In case of p-n junction diodes, the current flowing through the diode is not proportional to the pot. applied across the diode.



Thyristors: $\rightarrow$

In case of thyristors the graph b/w V and I is non linear. In certain positions there are more than one value of current for the same value of applied pot. Even for the region AB the value of current decreases with increase in pot. applied.



Resistivity $\rightarrow$

We know that

$$R \propto l$$

(length of the conductor)

$$\propto \frac{1}{A}$$

(A is area of cross-section of conductor.)

$$\Rightarrow R \propto \frac{l}{A}$$

$$\Rightarrow \boxed{R = \rho \frac{l}{A}}$$

where ρ is resistivity of the conductor and its value depends upon the nature of the conductor.

If $l = 1\text{m}$ and $A = 1\text{m}^2$

then $R = \rho \times \frac{l}{A} \Rightarrow R = \rho$.

Thus resistivity or specific resistance of the conductor is equal to the resistance of the conductor of unit length and unit area of cross-section.

Units of resistivity :-

we know that

$$R = \rho \frac{l}{A}$$

$$\Rightarrow \rho = \frac{RA}{l} = \frac{\text{ohm} \times \text{m}^2}{\text{m}} = \text{ohm metre } (\Omega \text{m})$$

Factors affecting electrical resistivity :-

Deduction of Ohm's Law :-

We know that, when electric field E is applied across the conductor of length l and area of cross-section A , then drift velocity is given as,

$$v_d = \frac{eE\tau}{m} \quad \text{--- (1) (mag. only)}$$

Also

$$I = neAv_d \quad \text{--- (mag only)}$$

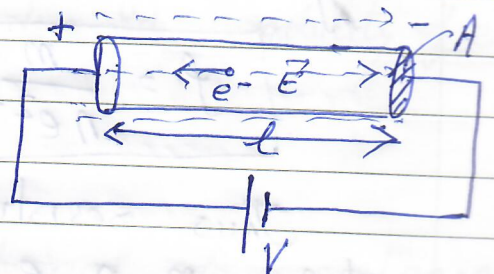
$$\Rightarrow v_d = \frac{I}{neA} \quad \text{--- (2)}$$

from (1) and (2).

$$\frac{eE\tau}{m} = \frac{I}{neA}$$

$$\Rightarrow \frac{eV}{l} \frac{\tau}{m} = \frac{I}{neA} \quad \left\{ \because V = Ed \right\}$$

$$\Rightarrow \frac{V}{I} = \frac{m}{ne^2\tau} \times \frac{l}{A} \quad \text{--- (3)}$$



for a given conductor mass of electron (m), charge on electron (e), no. of free e^- 's per unit volume (n),

avg. relaxation time (τ), length (l) and area of cross-section (A) of the conductor ~~is~~ is con

$$\therefore \frac{V}{I} = \text{Const.}$$

$$\Rightarrow V = \text{Const} \times I \quad \text{--- (3)}$$

$\Rightarrow V \propto I$ which is Ohm's law
Hence derived.

$$\text{Also } V = RI \quad \text{--- (4)}$$

$$\Rightarrow \frac{V}{I} = R \quad \text{--- (4)}$$

from (3) and (4)

$$R = \frac{m}{ne^2\tau} \frac{l}{A} \quad \text{--- (5)}$$

Thus resistance of the conductor depends upon m, n, e, τ (nature of the conductor) and l, A of the conductor.

$$\text{Also } R = \rho \frac{l}{A} \quad \text{--- (6)}$$

from (5) and (6)

$$\rho = \frac{m}{ne^2\tau}$$

Thus resistivity of the conductor depends upon m, n, e, τ i.e. since n and τ varies from conductor to conductor. Therefore resistivity of the conductor is diff. for diff. conductors.

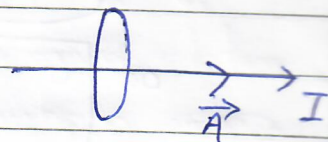
Resistivity in terms of current density $\Rightarrow$ We know that current density is d/o the value of current flowing through the conductor per unit area held normal to the flow of current.

$$\text{ie } j = \frac{I}{A}$$

$$= \frac{ne v_d A}{A}$$

$$\boxed{j = ne v_d}$$

$$= ne \left(\frac{eE\tau}{m} \right)$$



$$\left\{ \because v_d = \frac{eE\tau}{m} \right.$$

$$\Rightarrow j = \frac{ne^2\tau}{m} E$$

$$\Rightarrow j = \frac{E}{\rho}$$

$$\left\{ \because \rho = \frac{m}{ne^2\tau} \right\}$$

$$\therefore \boxed{\rho = \frac{E}{j}}$$

$$\text{or } \rho = \frac{E}{ne v_d}$$

$$\Rightarrow \boxed{\rho = \frac{1}{ne\mu}}$$

$$\text{where } \mu = \frac{v_d}{E} = \text{mobility of } e^-$$

Conductance and conductivity $\Rightarrow$

The reciprocal of resistance of the conductor is called its conductance.

$$\text{ie } G = \frac{1}{R}$$

$$\text{units } G = \frac{1}{\text{ohm}} = \text{ohm}^{-1} = \text{mho} = \Omega = \text{siemen}$$

The reciprocal of resistivity of the conductor is called conductivity of the conductor.

$$\sigma = \frac{1}{\rho}$$

$$\text{or } \sigma = \frac{J}{E} \Rightarrow J = \sigma E$$

$$\text{or } \sigma = ne\mu$$

Units

$$\sigma = \frac{1}{\rho} = \frac{1}{\text{ohm metre}} = \text{ohm}^{-1} \text{m}^{-1} = \text{Siemen m}^{-1} = \text{Sm}^{-1}$$

Temperature dependence of resistance and resistivity $\rightarrow$

We know that

$$\rho = \frac{m}{ne^2\tau}$$

$$\text{or } R = \frac{m}{ne^2\tau} \frac{l}{A}$$

for a metallic conductor no. of free e^- per unit volume is almost ~~constant~~ independent of temp. Thus resistivity or resistance of the is inversely proportional to avg. ~~relation~~ relaxation time.

$$\text{ie } \rho \propto \frac{1}{\tau}$$

$$\text{or } R \propto \frac{1}{\tau}$$

with increase in temp, velocity of the free e^- increase, therefore avg. relaxation time (time b/w two successive collisions) decreases. Hence resistivity or resistance of the metallic conductor increases linearly with increase in temp.

$$\text{ie } \boxed{\rho \propto T}$$

$$\text{or } \boxed{R \propto T}$$

But in case of semiconductors or insulators

The no. of free e^- 's per unit volume increases exponentially with increase in temp. Since resistivity is inversely proportional to n , therefore resistivity decreases exponentially with increase in temp.

$$\text{ie } \rho(T) = \rho_0 e^{E_g/kT}$$

for metallic conductors

$$R = R_0 (1 + \alpha \Delta T) \quad ; \quad \text{or} \quad \rho = \rho_0 (1 + \alpha \Delta T)$$

$$\Rightarrow R - R_0 = \alpha R_0 \Delta T \quad | \quad \text{or} \quad \rho - \rho_0 = \alpha \rho_0 \Delta T$$

$$\Rightarrow \alpha = \frac{R - R_0}{R_0 \Delta T}$$

$$\alpha = \frac{\rho - \rho_0}{\rho_0 \Delta T}$$

$$\Rightarrow \alpha = \frac{\Delta R}{R_0 \Delta T}$$

$$\Rightarrow \alpha = \frac{\Delta \rho}{\rho_0 \Delta T}$$

Thus temp. coeff. of resistance (or resistivity) (α) is defined as the change in resistance (or resistivity) of the conductor per unit resistance (or resistivity) per unit change in temp. of the conductor.

Note $\Rightarrow$

Alloys like constantan or manganin are used to make standard resistance coils because

- (1) they have very high value of resistivity and
- (2) they have very small value of temp. coeff. i.e. variation in resistivity / resistance with temp. for these alloys is very small (negligible)

Super conductivity $\Rightarrow$

The phenomenon of complete loss of resistance of the metal or alloy by lowering its temperature below a certain value is called superconductivity.

The temp. at which resistance/resistivity of the metal or alloy becomes zero is called critical temp.

ie at critical temp., ~~metal~~ resistance of the metal becomes zero and conductor becomes a super conductor.

Critical temp. for Hg is 4.2 K

" " " $YBa_2Cu_3O_7$ is 90 K

" " " $Tl_2Ca_2Ba_2Cu_3O_{10}$ is 120 K.

Scientists are working to find out an alloy whose critical temp is equal to room temp ie 300 K. Because current once set up in a super conductor loop can persist for years without any applied emf.

Heat loss ($H = I^2 R t$) in long distance power transmission will become zero over the wires made up of super conducting materials.

Cause of super conductivity $\rightarrow$ It is believed that at critical temp, all the ionic vibrations cease and hence e^- 's move ^{in pairs} towards the +ve terminal without being deflected by the ions. Thus resistance of the conductor becomes zero and it becomes a super conductor.

Colour coding of carbon resistors $\rightarrow$

The value of high resistant carbon resistor is marked on them in the form of colour codes.

A carbon resistor has usually four rings.

different colours. Each colour has a particular code from 0 to 9. A line BB ROY of GB has VGTW

BB ROY of Great Britain has Very Good Wife helps us to remember the codes.

Black	Brown	Red	Orange	Yellow	Green	Blue	Violet	Grey	White
0	1	2	3	4	5	6	7	8	9

Colours of the first ring gives 1st significant digit.

" " " 2nd " " 2nd " "

" " " 3rd " " no of zero that will follow

" " " 4th " " %age variation / % accuracy or %age tolerance.

4th ring is either of silver colour or of golden colour or there is no fourth ring (ie no colour)

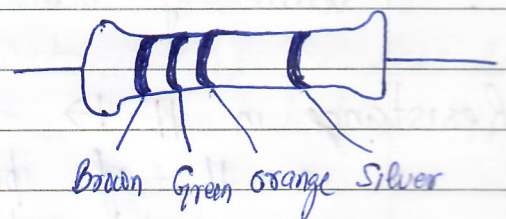
Golden coloured ring indicates $\pm 5\%$ variation/accuracy.

Silver " " " $\pm 10\%$ " / "

No " " " $\pm 20\%$ " / "

eg. for a carbon resistor Brown, green, orange and silver coloured rings the value of resistance will be

$$R = 15 \times 10^3 \pm 10\%$$



Resistors in series $\rightarrow$

Resistances are said to be connected in series if they are connected end to end so that same current flow through them.

Consider three resistors having resistances R_1 , R_2 and R_3 are connected in series across a battery of pot. diff. V volts as shown.

Since same current flows through all the

resistances, therefore pot. drop across each resistor can be given as

$$V_1 = IR_1 \quad \text{--- (1) (ohm's law)}$$

$$\text{Similarly } V_2 = IR_2 \quad \text{--- (2)}$$

$$V_3 = IR_3 \quad \text{--- (3)}$$

If R_s is the equivalent resistance of the series combination then

$$V = IR_s \quad \text{--- (4)}$$

Also

$$V = V_1 + V_2 + V_3$$

$$\Rightarrow IR_s = IR_1 + IR_2 + IR_3$$

{ using (1), (2), (3) and (4)}

$$\Rightarrow \boxed{R_s = R_1 + R_2 + R_3}$$

Equivalent resistance is more than the largest value of resistance connected in series.

Resistance in || $\rightarrow$ Resistances are said to be connected in || if pot. drop difference across each resistor is same and is equal to applied potential. Consider three resistors having resistances R_1 , R_2 and R_3 are connected in || across a battery of pot. diff. V volts.

~~Since resistances are connected in ||~~

Since pot. diff. across each resistance is same, therefore different currents depending on the value of resistance will flow through them.

$$\therefore I_1 = \frac{V}{R_1} \quad \text{--- (1)}$$

{ ohms law }

$$\text{Similarly } I_2 = \frac{V}{R_2} \quad \text{--- (2)}$$

$$\therefore I_3 = \frac{V}{R_3} \quad \text{--- (3)}$$

If R_p is the equivalent resistance of the || combination then

$$I = \frac{V}{R_p} \quad \text{--- (4)}$$

Also

$$I = I_1 + I_2 + I_3$$

$$\Rightarrow \frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

{ using (1), (2), (3) & (4) }

$$\Rightarrow \boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

The value of equivalent resistance R_p is less than the smallest value of resistance connected in ||.

Note $\Rightarrow$

for two resistance R_1, R_2 connected in ||.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_p} = \frac{R_2 + R_1}{R_1 R_2}$$

$$\Rightarrow \boxed{R_p = \frac{R_1 R_2}{R_1 + R_2}}$$

$\Rightarrow$

$$\boxed{R_p = \frac{\text{Product of two}}{\text{Sum of two}}}$$

Internal Resistance of the cell $\rightarrow$ The resistance offered by the electrolyte of the cell ~~is called~~ to the flow of electric current b/w its electrodes is called internal resistance of the cell.

It depends upon:

- (1) Nature of the electrolyte and its concentration.
2. It is directly $\propto$ to the distance b/w electrodes.
3. It is inversely $\propto$ to area of the electrodes immersed in the electrolyte.

Electromotive force or emf of the cell $\rightarrow$ The pot. diff. between the two terminals of the cell in an open circuit (when no current is drawn from the cell) is called emf of the cell. (E)

It is also d/d the work done to drive a unit charge round the complete circuit.

emf of the cell is said to be 1 Volt if 1 J of work is done by the cell to drive 1 C charge round the circuit.

Also emf is not a force but it is work done per unit charge.

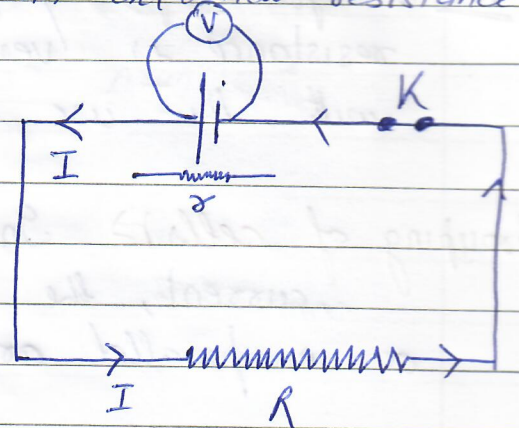
Terminal potential Difference $\rightarrow$ The pot. diff. b/w the terminals of the cell in a closed circuit (when current is drawn from the cell) is called terminal pot. diff. of the cell. (V)

Expression for internal resistance of the cell $\rightarrow$

Consider a cell of EMF E and internal resistance r .

resistance r is connected to an external resistance R and key K as shown.

When key is kept open i.e. no current is drawn from the cell, voltmeter reading gives the emf of the cell E .



When key K is closed, the entire emf of the cell gets dropped across the ext. resistance R and internal resistance r of the cell.

Thus $E = \text{Pot. drop across } R + \text{Pot. drop across } r$.

$$\Rightarrow E = IR + Ir$$

$$\Rightarrow E = I(R + r)$$

$$\Rightarrow I = \frac{E}{R + r}$$

————— ①

$$\begin{aligned} E &= Ir + IR \\ V &= IR \\ V &= I \end{aligned}$$

Also when key is closed, the voltmeter reads the pot. drop across ext. resistance R i.e. term. pot. diff. of the cell.

$$\text{i.e. } V = IR$$

$$\Rightarrow V = \frac{E}{R + r} R$$

{using ①}

$$\Rightarrow R + r = \frac{E}{V} R$$

$$\Rightarrow r = \frac{E}{V} R - R$$

$$\Rightarrow \boxed{r = \left(\frac{E}{V} - 1 \right) R}$$

$$\boxed{r = \left(\frac{E - V}{V} \right) R}$$

Note $\Rightarrow$ for a freshly prepared cell, the value of internal resistance is very low but it goes on increasing with the use of cell.

coupling of cells $\Rightarrow$ In order to have larger value of current, the cells are usually connected in series or in parallel or are mixed grouped.

~~cells in series~~ $\Rightarrow$ When cells are connected in series. then equivalent emf of the cells and equivalent internal resistance of the combination can be given as

$$E_{eq} = E_1 + E_2 + \dots + E_n$$

$$\text{and } r_{eq} = r_1 + r_2 + \dots + r_n$$

refer to S.L. Arora
Shanpat Rai & Sons
page no - 3.51 - 3.52

If all cells are identical of emf E and internal resistance r then

$$E_{eq} = E + E + \dots \text{ n times}$$

$$\text{ie } E_{eq} = nE$$

$$\text{and } r_{eq} = nr$$

When cells are connected in parallel, then equivalent emf and int. resistance of the combination can be given as,

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \dots + \frac{E_n}{r_n}$$

$$\text{and } \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}$$

refer to S.L. Arora
Shanpat Rai & Sons
page no - 3.52 - 3

If all the cells are identical

then $\frac{1}{r_{eq}} = \frac{1}{r} + \frac{1}{r} + \dots$ n times

$$\Rightarrow \frac{1}{r_{eq}} = \frac{n}{r}$$

$$\therefore \boxed{r_{eq} = \frac{r}{n}}$$

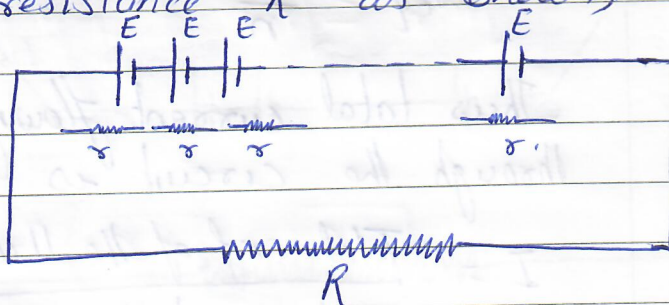
and $\frac{E_{eq}}{r_{eq}} = \frac{E}{r} + \frac{E}{r} + \dots$ n times

$$\Rightarrow \frac{E_{eq}}{r/n} = \frac{nE}{r}$$

$$\Rightarrow \boxed{E_{eq} = E}$$

Cells in series $\rightarrow$ Consider n cells of emf E each and internal resistance r each are connected in series across an external resistance R as shown,

Thus total current flowing through the circuit is given as



$$I = \frac{\text{Total emf of the combination}}{\text{Total resistance of the circuit.}}$$

$$\Rightarrow \boxed{I = \frac{nE}{R + nr}}$$

Sp. Cases $\rightarrow$

① when $R \ll nr$

then $I = \frac{nE}{nr}$

$$\Rightarrow I = \frac{E}{r} = \text{Current due to a single cell.}$$

② when $R \gg n\tau$

then $I = \frac{nE}{R} = n$ times the current due to a single cell

Thus for a heavy external load (R), cells are connected in series to have larger current through circuit.

Cells in // $\Rightarrow$ Consider n cells each of EMF E and internal resistance τ are connected in // across an external load resistance R as shown. We know that for a // combination

Total emf of the combination

$$E_{eq} = E$$

and total int. resistance of the combination

$$\tau_{eq} = \frac{\tau}{n}$$

Thus total current flowing through the circuit is

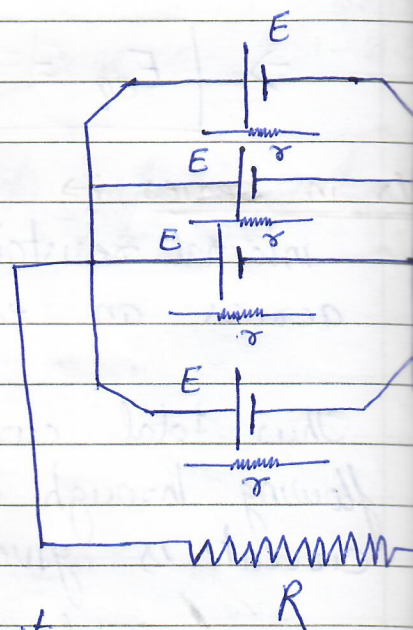
$$I = \frac{\text{Total emf of the // combination}}{\text{Total resistance of the circuit}}$$

$$\Rightarrow \boxed{I = \frac{E}{R + \frac{\tau}{n}}}$$

Sp. cases

① when $R \ll \tau$

then $I = \frac{E}{\tau/n} = n \frac{E}{\tau} = n$ times current due to a single cell



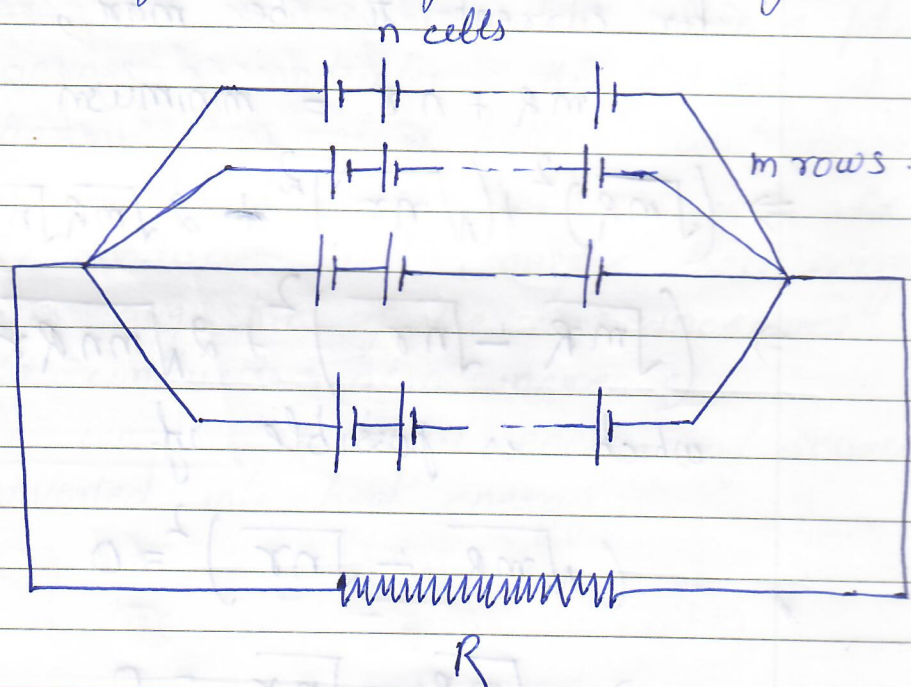
② for $R \gg r$

$$I = \frac{E}{R} = \text{current due to a single cell.}$$

Thus for a small value of load resistance (R), cells are connected in || to have larger current through the circuit.

Mixed grouping of the cells $\rightarrow$

If load resistance is of the order of internal resistance of the combination then mixed grouping is done to have larger value of current through the circuit.



Consider a number of cells each of emf E and internal resistance r are connected in mixed grouping having m rows and each row having n cells as shown,

Thus total emf of the combination.

$$E_{eq} = nE$$

and total int. resistance of the combination

$$r_{eq} = \frac{nr}{m}$$

Thus total current flowing through the circuit is

classmate

$$I = \frac{\text{Total emf}}{\text{Total resistance}} \Rightarrow I = \frac{nE}{R + \frac{nr}{m}}$$

$$\Rightarrow I = \frac{nE}{\frac{mR + nR}{m}}$$

$$\Rightarrow I = \frac{mnE}{mR + nR}$$

$$\Rightarrow \boxed{I = \frac{NE}{mR + nR}}$$

$$\left\{ \because mn = N \right.$$

$$\Rightarrow \boxed{I = \frac{NE}{\text{rows time } R + \text{columns time } R}}$$

for current to be max,

$$mR + nR = \text{minimum}$$

$$\Rightarrow (\sqrt{mR})^2 + (\sqrt{nR})^2 + 2\sqrt{mR}\sqrt{nR} + 2\sqrt{mR}\sqrt{nR} = \text{min}$$

$$\Rightarrow (\sqrt{mR} - \sqrt{nR})^2 + 2\sqrt{mnR^2} = \text{min.}$$

which is possible if

$$(\sqrt{mR} - \sqrt{nR})^2 = 0$$

$$\Rightarrow \sqrt{mR} - \sqrt{nR} = 0$$

$$\Rightarrow \sqrt{mR} = \sqrt{nR}$$

$$\Rightarrow mR = nR$$

$$\Rightarrow R = \frac{nR}{m}$$

$$\Rightarrow \boxed{R = R_{eq}}$$

classmate

Thus in mixed grouping, current through the circuit will be max when load resistance is equal to the total int. resistance of the circuit.